Appendix C

Spectroscopic Ellipsometry

Spectroscopic ellipsometry is a thin film optical characterization technique that sends s- and p-
linearly polarized light onto layered thin films and measures the light that is reflected back. By
doing so, the technique is able to determine the thickness of the individual layers as well as their
complex optical constants, $\tilde{n} = n + i\kappa$.

More specifically, the s- and p-polarized light reflects from each layer in the film stack based on
the Fresnel equations. For a single layer, these are given as:

$$R_{s,p} = \frac{r_{s,p}^{12} + r_{s,p}^{23}e^{-\beta}}{1 + r_{s,p}^{12}r_{s,p}^{23}e^{-2\beta}}$$  \hspace{1cm} (C.1)

where $r_{s,p}^{ab}$ corresponds to the reflection coefficient from the “ab” interface for s- and p-polarized
light, respectively. To relate these coefficients to the complex index of refraction for the layer,
$\tilde{n} = n + i\kappa$, we look at $\beta$:

$$\beta = 2\pi \left( \frac{d}{\lambda} \right) n_2(\lambda)\cos(\phi_2)$$  \hspace{1cm} (C.2)

where $d$ is the thickness of layer 2, $\lambda$ is the wavelength at which the measurement was taken, $n_2(\lambda)$
is the complex index of refraction of layer 2 at $\lambda$, and $\phi$ is the phase shift that results from the s-
or p-polarized light propagating through the layer.

By measuring $R_{s,p}$ and $\beta$, as well as the change in phase between the incident light and reflected
light for both polarizations, two parameters can be calculated:

$$\Psi = \left| \frac{R_p}{R_s} \right|$$  \hspace{1cm} (C.3)

$$\Delta = \delta_1 - \delta_2$$  \hspace{1cm} (C.4)
where $\delta_{1,2}$ are the differences in phase between the s- and p-polarized light before (1) and after (2) propagating through the layer.

Figure C.1. Shown here is layout for spectroscopic ellipsometry [4] which shows: both s- and p-polarized light incident on the sample surface, the angle of incidence, $\phi$; the difference in phase differences, $\Delta$; and the ratio of $R_p$ and $R_s$: $\Psi$.

Each layer in the film stack is mathematically represented by an appropriate model which relates the thickness and dielectric constants of the material. The model for the entire stack is plotted against the measured $\Psi$ and $\Delta$ data. By measuring these values over a range of wavelengths and angles of incidence, a minimization routine is able to fit the experimental data to the models. Once an acceptable fit is achieved, the models for each layer can be used to extract the dispersion of the materials being studied: $\tilde{n}(\lambda) = n(\lambda) + i\kappa(\lambda)$.

As an example, ellipsometry data for lithium niobate that was used in this thesis was modeled using a Cauchy layer fit which is an empirical model for ellipsometry data. As was stated in Chapter 6, the data was fit from 400 - 700 nm using:

\[ n(\lambda) = n_0 + C_0 \frac{n_1}{\lambda^2} + C_1 \frac{n_2}{\lambda^4} \]  
\[ k(\lambda) = k_0 + C_0 \frac{k_1}{\lambda^2} + C_1 \frac{k_2}{\lambda^4} \]  

where $n_0 = 2.093$, $n_1 = 305.9$, $n_2 = 52.9$, $k_0 = 0.02$, $k_1 = 24.541$, $k_2 = 23.276$, $C_0 = 10^2$, $C_1 = 10^{133}$.
$C_1 = 10^7$, and $\lambda$ is given in nm.

In contrast, the indium tin oxide discussed in Chapter 8 was modeled using the drude model which is a physical model given by:

$$\epsilon = \epsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\omega\tau}$$  \hfill (C.7)

$$\omega_p = \frac{ne^2}{\epsilon m^*}$$  \hfill (C.8)

where the parameters $m^* = 0.45m_e$, $\epsilon_\infty = 4.2$, $\tau = 1 \text{ fs}$, $\lambda_p = 2300 \text{ nm}$, and $n = 4 \times 10^{20} \text{ cm}^{-3}$ were extracted using ellipsometry. Shown in Figure C.2 is an example of the $\Psi$ and $\Delta$ data that was taken for a thin film stack of Au/InTiO$_3$/SiO$_2$/Au across the visible spectrum with and without an applied voltage.

![Figure C.2. The ellipsometry measurements of the thin film stack shown in Figure 8.2.](image-url)