MODEL-BASED DECISION MAKING
IN THE HUMAN BRAIN

Thesis by
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In Partial Fulfillment of the Requirements
for the Degree of

Doctor of Philosophy

CALIFORNIA INSTITUTE OF TECHNOLOGY

Pasadena, California

2007
(Defended May 17th, 2007)
I would like to thank my thesis advisor and mentor John O’Doherty. I came to his lab with a passion for science, and an unquenchable curiosity in the most diverse questions about the functioning of the brain. He not only directed that energy into fruitful endeavors and provided key insights along the way, but also taught me how to know when enough research on a given topic had been carried out, how to present the results centered on the relevant questions, and how to expand the horizon of questions asked given the current research. In other words, the final sculpting chisels that form a researcher, and lessons many do not get but through years of experience. Research in John’s lab was not only productive, it was also eye opening, collegial, and fun.

I also want to thank my collaborator and mentor Peter Bossaerts. With him, John and I worked on the more complex modeling aspects in this thesis, for which he had a never-ending stream of insightful advice and groundbreaking ideas. With him I also explored the implications of risk in human choice, and more generally tied reward-based decision making with the nascent field of Neuroeconomics. I also want to thank the rest of my thesis committee: Ralph Adolphs, Shin Shimojo and Steve Quartz for their support, advice and availability.

During my first two years at Caltech, I worked with Chris Adami and collaborated with Pietro Perona. Although work carried out during this time is not included in this thesis, the questions on Bayesian unsupervised learning I explored then have had a direct and unquestionable impact in my research. I am most grateful to Chris for letting me follow my ideas to the very end. Without his support, this thesis would not have come to be.
More generally, I want to thank people that, although I did not collaborate directly with, formed the positive research environment for work in reward based decision making that forms the backdrop of the Caltech Brain Imaging Center. Thanks Antonio Rangel and Colin Camerer from HSS, for always being open to questions from people out of their labs, and Mike Tyszka, Michael Spezio for advice. Thanks Vivian Valentin, Saori Tanaka, Signe Bray, Hackjin Kim, Elizabeth Tricomi and Klaus Wunderlich in John’s lab; Kerstin Preuschoff, Tony Bruguier, Ulrik Beierholm, Jan Glaescher, Dirk Neumann, Asha Iyer, Axel Lindner, Igor Kagan, Hillary Glidden, Hilke Plassmann, Nao Tsuchiya, Jessica Edwards, Meghana Bhatt, Cedric Anen for fruitful discussions, code sharing, insights and friendship. Also thanks to Steve Flaherty, Mary Martin and Ralph Lee for making the CBIC work on wheels.

I also have many friends at Caltech who have made these PhD years pass by with a colorful tone and who, unbeknownst to them, contributed indirectly to this work by answering questions relevant to their particular expertise, providing support, interesting side discussions and creating a fruitful venue to relax. Thanks Grant Mulliken, Allan Drummond, Kai Shen, and David Soloveichik, with whom I started the CNS program at Caltech; Alex Holub, Fady Hajjar, Guillaume Brès, Lisa Goggin, Hannes Helgason, Stephane Lintner, Angel Angulo, Daven Henze, Elena Hartoonian, Corinne Ladous, Setthivoine You, Alex Gagnon, Jesse Bloom, Vala Hjörleifsdóttir, and Michael Campos. I also want to thank friends outside of Caltech, who contributed to this work in pretty much the same way.

Last of all, I want to thank my parents and sisters for the support and patience they have given during the years. Much of what I am now I owe to them. And to my dearest Allison, for her love and patience, and for knowing how to encourage me when things got tough, and tone me down when I was too excited!
ABSTRACT

Many real-life decision making problems incorporate higher-order structure, involving interdependencies between different stimuli, actions, and subsequent rewards. It is not known whether brain regions implicated in decision making, such as ventromedial prefrontal cortex, employ a stored model of the task structure to guide choice (model-based decision making) or merely learn action or state values without assuming higher-order structure, as in standard reinforcement learning. To discriminate between these possibilities we scanned human subjects with fMRI while they performed two different decision making tasks with higher-order structure: probabilistic reversal learning, in which subjects had to infer which of two choices was the more rewarding and then flexibly switch their choice when contingencies changed; and the inspection game, in which subjects had to successfully compete against an intelligent adversary by mentalizing the opponent’s state of mind in order to anticipate the opponent’s behavior in future. For both tasks we found that neural activity in a key decision making region: ventromedial prefrontal cortex, was more consistent with computational models that exploit higher-order structure, than with simple reinforcement learning. Moreover, in the social interaction game, subjects were found to employ a sophisticated strategy whereby they used knowledge of how their actions would influence the actions of their opponent to guide their choices. Specific computational signals required for the implementation of such a strategy were present in medial prefrontal cortex and superior temporal sulcus, providing insight into the basic computations underlying competitive strategic interactions. These results suggest that brain regions such as ventromedial prefrontal cortex employ an abstract model of task structure to guide behavioral choice, computations that may underlie the human capacity for complex social interactions and abstract strategizing.
# TABLE OF CONTENTS

Acknowledgements ............................................................................................ iii  
Abstract ................................................................................................................ v  
Table of Contents................................................................................................ vi  
List of Illustrations............................................................................................. vii  
List of Tables ...................................................................................................... ix  
Nomenclature ....................................................................................................... x  

*Chapter 1*: Introduction ....................................................................................... 1  

*Chapter 2*: Model-based Decision Making in Humans................................. 12  

*Chapter 3*: Predicting Behavioral Decisions with fMRI.................................. 52  

*Chapter 4*: Amygdala Contributions................................................................. 95  

*Chapter 5*: Thinking of You Thinking of Me................................................. 134  

References ........................................................................................................ 160  

*Appendix A*: Bayesian Inference ..................................................................... 175  

*Appendix B*: Inference Dynamical Equivalents............................................. 185
LIST OF ILLUSTRATIONS

Number Page

2.1. Reversal task setup and state-based decision model ..................................................31
2.2. Correct choice prior, and posterior - prior update signals in the brain ......................33
2.3. Standard RL and abstract state-based decision models make qualitatively different predictions about the brain activity after subjects switch their choice. ......35
2.4. Incorrect choice prior and switch - stay signals in the brain ......................................37
S2.1. Comparison of the behavioral fits of the state-based decision model to a variety of standard Reinforcement Learning algorithms ...........................................44
S2.2. Behavioral data and model predictions for three randomly chosen subjects.............46
S2.3. Plot of the model-predicted choice probabilities derived from the best-fitting RL algorithm before and after subjects switch their choice. ................................. 48
3.1. Reversal task setup and multivariate classifier construction......................................68
3.2. Global and local fMRI signals related to behavioral choice ......................................71
3.3. Illustration of the decoding accuracy for subjects’ subsequent behavioral choices for each individual region and combination across regions..........................73
S3.1. Brain regions of interest .......................................................................................... 77
S3.2. Comparison of fMRI classifiers ..............................................................................79
S3.3. Switch vs. stay for individual subjects.......................................................................81
S3.4. Normalized cross-correlation of regression residuals across regions ....................83
S3.5. Classification Receiver Operating Characteristic Curve ......................................85
S3.6. Behavioral choice responses and the signal-to-noise ratio in each region of interest.......................................................................................................................87
S3.7. Regional classifiers .................................................................................................89
4.1. Axial T1-weighted structural MR images from the two amygdala lesion subjects
4.2. Probabilistic reversal task
4.3. Behavioral performance
4.4. Behavioral choice signals
4.5. Expected reward signals in the brain
4.6. Responses to receipt of rewarding and punishing outcomes
S4.1. Subject performance during task acquisition
S4.2. Controlling for the effects of behavioral differences between amygdala subjects and controls on signals pertaining to expected reward value
S4.3. Controlling for behavioral differences between amygdala lesion subjects and controls in signals pertaining to behavioral choice
S4.4. Multiple axial slices for both amygdala lesion subjects
5.1. Inspection game and behavioral results
5.2. Expected reward signals
5.3. Influence signals in the brain
S5.1. Out-of-sample model log likelihood
S5.2. Model comparisons with respect to the processing of expected reward signals in the brain
S5.3. Prediction error signals
A.1. Conditional probability distributions of y given a fixed value of the random variable x
A.2. Directed graphical model linking six random variables
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Number</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2.1.  Mean parameters across subjects for the behavioral models</td>
<td>50</td>
</tr>
<tr>
<td>S2.2.  fMRI activity localization</td>
<td>51</td>
</tr>
<tr>
<td>S3.1.  Multivariate classifier accuracy in decoding choice across subjects and subject sessions</td>
<td>91</td>
</tr>
<tr>
<td>S3.2.  Multivariate classifier ROC areas across subjects and subject sessions</td>
<td>92</td>
</tr>
<tr>
<td>S3.3.  Regions of interest</td>
<td>93</td>
</tr>
<tr>
<td>5.1.   Model update rules</td>
<td>147</td>
</tr>
</tbody>
</table>
NOMENCLATURE

**ACC** – Anterior Cingulate Cortex

**BOLD signal** – Blood Oxygenation Level Dependent signal

**DLPFC** – Dorso-Lateral Prefrontal Cortex

**fMRI** – Functional Magnetic Resonance Imaging

**HMM** – Hidden Markov Model

**PFC** – Prefrontal Cortex

**STS** – Superior Temporal Sulcus

**OFC** – Orbitofrontal Cortex
Chapter 1

INTRODUCTION

Prelude

From an evolutionary perspective, rewards and punishments could be considered as events that heighten or lower the probability of an organism reproducing. The identification of what constitutes a rewarding and punishing event for a particular organism can be considered to be genetically hardwired, given that the evaluation of the final objective of reproductive efficiency is accomplished across various organism generations. These constitute primary rewards such as food and sex, and primary punishments such as physical pain; all leading directly or indirectly to heightening or lowering an organism’s survival and reproduction. Moreover, during the life of an organism, events that are associated to these primary rewards and punishments can be learnt.

The first organism in which associative learning (classical conditioning) was extensively studied was with *Aplysia’s* (a sea hare, shell-less mollusk) siphon-withdrawal reflex. One defense mechanism that the organism evolved is to withdraw its siphon when it is stimulated. Stimulation of the tail has no such effect on the siphon. However, after repeated stimulation of the tail followed shortly by stimulation of the siphon, tail stimulation would eventually induce siphon withdrawal on its own. The mechanism by which neurons learn to associate coincident events was first postulated by Donald Hebb, and basically states that the more frequently a pre-synaptic neuron fires coincidently with the firing of a post-synaptic neuron, the higher the probability that the pre-synaptic neuron will lead to the firing of the post-synaptic neuron. This type of learning involves a modification of the

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i With higher level organisms that have social customs that can be passed on from generation to generation, a second form of evolution takes place, with the survival of customs that indirectly lead to the survival and reproduction of the organisms that embrace them. Thus, social rewards and punishments are not genetically hardwired, but learnt during the lifetime of an organism, and delivered by the rest of society.
synaptic weight, or connectivity, between two neurons through a series of biochemical reactions at the synaptic site\textsuperscript{4,5}.

Higher cognitive skills\textsuperscript{6} basically extend this ability of associating complex external stimuli with primary rewards and punishments. For example, the visual system can distinguish a snake from a log (recognition–ventral stream), and understand the movement of objects (dorsal stream) that lead to upcoming rewarding or punishing events, all from the pattern of environment light hitting the retina. Furthermore, organisms have a great degree of freedom in making choices, and many times have multiple, competing predicted rewards and punishments that could be a consequence of a decision. Thus, the association of external stimuli with primary rewarding and punishing events has to be condensed into a final abstract internal value (or utility) to guide decisions. It is from here I will start my introduction, going quickly over the literature on how organisms might computationally assign an abstract value (or reward expectation) so as to guide choice in complex, real-world environments, and start to tease apart the brain structures that execute different components of these algorithms in the human brain.

**States and Decisions**

The problem to be solved by an organism is to make decisions so as to maximize the reward, or utility, it obtains. This problem can be broken down into separate components which will be addressed shortly. Before we do so, I will present a common framework, or paradigm, to better understand the problem itself. A mechanical view of the world is one in which the whole environment (including the organism itself \textit{in} the environment) can be characterized as a uniquely identifiable state at any point in time, and that the flow of time is simply having the world jump (transition) from state to state. Having the ability to make a decision implies that this state flow can be influenced such that the environment (plus organism) ends up in a given state contingent on the decision made. A simple example is when a person is in front of two doors. The state he/she is in can be titled ‘in front of two doors,’ and depending on what door he/she chooses, the ‘left’ or ‘right’ door, he/she will end in one of two alternative states, i.e., ‘rooms.’
The problem of making decisions so as to maximize rewards can be broken down into:

1. **State categorization**: what makes one state different from another

2. **State identification**: which state an organism is in

3. **State value**: the reward/punishment an organism receives as a consequence of being in a given state

4. **State flow**: how states change (transition) with time

5. **Decisions**: how a decision affects the flow of states

Although the world may consist of uniquely identifiable states, an organism still has to learn and categorize them, leading to an internal representation of world states. This will not be directly addressed by work in this thesis. The best illustration of the more biologically oriented advances in the field can be found in research on how the visual system learns to categorize (and subsequently identify) objects and scenes\textsuperscript{7, 8} using unsupervised learning objectives such as sparse\textsuperscript{9} and predictive coding\textsuperscript{10}. Once an internal representation of state categories exists, an organism has to infer in what state it is currently in. Most reward learning research will assume that a state can be perfectly identified, whereas in real world scenarios an organism will have some uncertainty of which state it is in, leading to a probability distribution over states. This is the situation when things are hard to recognize due to environmental noise (Is it a snake or a log?), or when internal states are not well mapped to environmental states (representation noise). One objective of learning is to minimize the latter, and make the mapping as good as possible.

Moreover, there is a constraint on the number of states that can be encoded by a brain with limited capacity. A person from the tropics might define a certain state of water found in colder regions as ‘solid water,’ while a person from a more temperate region will differentiate between ‘snow’ and ‘ice.’ This can be defined as a coding problem\textsuperscript{11}. For an organism, states that happen more often in an environment, or that have a bigger variance in received rewards and punishments, are better differentiated in comparison to states that
do not happen often, or have very similar consequent rewards, which are lumped together and identified as one single state. Thus, learning how to encode environment states and the value of those states go hand in hand. Furthermore, it is also advantageous not to encode the whole environment as one single state (grandmother representation), but as best as possible as a combination of unique states that can happen independently at the same time (sparse representation). This is illustrated again with the visual system, in which one can see more than one object which might have different identities and assigned values, but that sometimes can be found together in the same scene independently from each other. However, we will assume the state representation of the environment is given \textit{a priori}, and follow up with the question of how values are assigned to existing states through experience.

With the passage of time, environment states follow one another with certain (physical) rules. Knowing or having an internal model of how states transform into each other is useful to predict what is to come, and what rewards to expect in future. The probability of the environment arriving at a given state may well depend on the history of all previous states that were visited beforehand. However, this can be relaxed somewhat in many problems such that the probability of jumping to a state only depends on the current state, and not on other states further back in history (i.e., a Markov process).

Last, and importantly, is the question of how decisions affect this process. An organism making a decision ceases to be an observer of the state flow of the environment, with the rewards it might receive in each state, but rather an active actor who can decide which state(s) will follow. That is, an organism’s decision defines the probability distribution of states to follow. This choice will be such so as to maximize the rewards received in the future, consequent on the environment states visited and decisions made thereafter. Moreover, organisms apply a discount on future rewards, in part due to their uncertainty (The type of discounting that humans apply to future monetary rewards is an area of contention in economic fields\textsuperscript{12,13}.)

Computational Reward Learning

Given an existing state representation of the environment, we will tease apart how rewards and decision policies (what decision to make at each state) are learned.

State Values

The most straightforward approach to learning state values is by sampling, in which the value of the state is updated given the reward the organism receives such that the value is equivalent to the immediate expected reward \( V_s = \langle R_s \rangle \). This is known as the Rescorla-Wagner rule\(^{14} \), in which the value of state \( S \) is updated with a prediction error, \( \delta = R - V_s \), between the expected value of the state \( V_s \) and the reward received \( R \):

\[
V_s = V_s + \eta \delta ,
\]

(1.1)

where \( \eta \) is the learning rate.

From the learned state values, the expectation of future rewards of any given state can be calculated by adding the value of that state and the value of future states visited thereafter contingent on the actual decision policy (future decisions), and the discount on future rewards. The expectation of future rewards then guide decision making. However, knowing \textit{a priori} all states and decisions that will be made in the future requires a precise model of state flow (including future decisions made) and tracing all possible future branches is computationally intensive and time consuming. An alternative, model-free approach is to sample the future expected reward of each state directly instead of their immediate expected reward. The future expected reward of a state can be recursively defined (Bellman equation\(^{15} \)) as the immediate expected reward of that state, plus the expected reward of future states (expectation over future states visited due to the current decision policy) reduced with a temporal discount:

\[
V_{s(t)} = \langle R_{s(t)} \rangle + \gamma \langle V_{s(t+1)} \rangle ,
\]

(1.2)
Thus, future expected rewards can be learned directly with a simple update rule, 
\[ \delta = R + \gamma V_{S(t+1)} - V_{S(t)} \], known as a temporal difference (TD) update\(^{16}\).

**Choice selection**

A simple way to decide between two future states is by choosing the one with the highest sampled value. However, if a state has only been sampled a few times, the value of the state might still be unknown, i.e., there is reward uncertainty. The implicit value of reducing state uncertainty when making decisions is known as the explore-exploit dilemma\(^ {17} \). The resulting choice stochasticity due to reward uncertainty can be approximated by a softmax distribution across choices\(^ {18} \), in which the probability \( P_s \) of choosing state \( S \) is bigger the higher its value \( V_S \), but where the probability of choosing other states is non-zero (controlled by a noise or exploration parameter \( 1/\beta \)):

\[
P_s = \frac{e^{\beta V_s}}{\sum_i e^{\beta V_i}}.
\] (1.3)

Furthermore, the reward associated with a given state might not be fixed, but rather a distribution of values. The most simple, perhaps, is for rewards to have a Gaussian distribution with a certain mean and variance, given a state. Thus, rewards might randomly be higher or lower than the mean. This is known as reward risk, and it biases how people make choices\(^ {19, 20} \). Humans not only maximize their expected reward when making decisions, but also take into consideration the riskiness of those rewards. Furthermore, humans exhibit risk preferences that change according to whether outcomes are perceived as gains or losses, something that arguably only depends on how those outcomes are measured relative to an arbitrary frame: an observation known as prospect theory\(^ {21} \). However, the algorithms presented in this introduction can easily be extended to use utility values that incorporate both the rewards received and their associated risks. The study of people’s behavior when making monetary decisions is part of the nascent field of Neuroeconomics\(^ {22, 23} \).
Bayesian forward models

As mentioned earlier, a model of state transitions can be constructed to estimate future expected rewards. In effect, this is a model of the environment dynamics, and it can be used to look forward at all possible outcomes in the future given the current state. If each state is also assigned a mean expected reward value (for immediate rewards), then this forward-looking process can be used to calculate the discounted expected value of that state by integrating over all future alternative paths. Crucially, forward models have two distinct components in comparison to sampling-based algorithms. First, the model of how environment states are linked to each other, and their immediate associated rewards, has to be learnt. Secondly, the current model can be used to guide decisions by inferring the state an organism is currently in, and estimating in a forward manner the associated expected rewards. These two steps can alternate with each other in what is usually referred to as EM (Expectation-Maximization), where the first refers to the inference step and the latter to the learning step (see Appendix A). Moreover, the inference step can be simplified into an expected value update equation similar to 1.1, but where all the state values are updated simultaneously as shown below:

$$V_S = V_S + \eta (R - V_S)$$

$$V_{S'} = V_{S'} + \eta (R_{\text{foregone}} - V_{S'})$$

where $S$ is the chosen state, and $S'$ are the un-chosen or foregone states (see Appendix B for a complete derivation). The relation between the rewards assigned to update each state not only depends on the outcome but also on the structure, or relation between states, of the model. These are not as simple as the reward that would have been received had the other state been chosen. Thus, it is important to point out that, although expected value update equivalents are intriguing because of their similarity to RL updating, they are just a proxy for the correct underlying interpretation: that of hidden state inference.

Summary

Decision making involves knowing the future expected rewards of possible states. This can be done in a forward model search approach, or in a model-free RL expected reward
sampling approach. The first is computationally intensive, time consuming, and assumes knowledge of the state flow of the environment; while the later is ignorant about environmental state flow, and computationally quick, but reaches optimal behavior slowly after extensive sampling of the environment. Furthermore, forward models can be quick to incorporate new rules and thus adapt to a changing environment, while RL models are quite slow in adapting to changing reward contingencies. An intermediate approach can be used, in which expected rewards are computed by taking a few steps into the future with a forward model, but replacing future steps with sample expected rewards. This combines the flexibility of forward models (in the short term) with the speed of sampling methods (for expected rewards further in the future). Likewise, both approaches could be implemented in parallel and used appropriately, depending on the circumstances\textsuperscript{25}.

**Brain Correlates**

The question of which of these algorithms is used by the human brain, and specifically, what brain structures execute different algorithmic components will now be addressed.

Historically, the first structure to be extensively studied in mammalian brains (rats, monkeys) was the hippocampus, due to its high neural density which made it easy to record from extra-cellularly. In particular, these neurons were found to quickly associate incoming signals (among others, leading to the formulation of the Hopfield network\textsuperscript{26} as a model of memory storage in the hippocampus), and to display a variety of adaptive behavior involving connectivity changes at the synaptic level. The hippocampus is thought to be the location of declarative associative memory formation\textsuperscript{27}, from which its contents are then also transferred to other neocortical structures\textsuperscript{28}. Patients with bilateral hippocampal lesions (from surgical ablation as a corrective measure for seizures, or prolonged alcohol abuse) cannot form new long term memories but find old memories relatively untouched\textsuperscript{29, 30}, depending on the damage extent. Thus, hippocampus can be thought of as an integral part in forming high-level state representations, by associating activities from different cortical regions.
Neural signals in different brain regions have been found to guide choice in a variety of contexts, from discriminating between noisy sensory stimuli, to choosing between stimuli depending on taste, physical pain, and monetary rewards. As discussed previously, a common reward currency (or utility) should be used to guide choice across these different modes of rewards and punishments. fMRI studies in humans, and neurophysiologic studies in rats and primates, have shown activity correlating with reward expectations across all modalities in mOFC, suggesting this area as encoding an abstract representation of reward for the guidance of choice. Moreover, there is evidence that the interaction between amygdala and mOFC is crucial for the generation of expected reward signals. Lesions in mOFC extending up the medial PFC wall have led to specific deficits in choice behavior: learning is unimpaired, but the ability to adapt to changing contingencies, such as in reversal learning, is diminished.

A key component of reinforcement learning algorithms is the formation of prediction errors, that is, the difference between rewards obtained and those expected. Schultz, Dayan, and Montague showed that the activity of dopamine neurons in substantia nigra encode reward prediction errors. Furthermore, they showed that this signal displayed the characteristics of a temporal prediction error in that reward expectations were progressively transferred to the earliest state that would predict that reward. Substantia nigra dopamine neurons mainly project to striatum and medial structures in mOFC, mPFC, and ACC. Imaging studies find BOLD activity to prediction errors in striatal structures, as well as temporal difference errors. The finding of neural structures encoding temporal difference errors in principle advocate the brain as implementing a model-free approach for state reward representations, in which future expected rewards are directly encoded for every environment state. However, these findings do not exclude a model-based approach, in which states would only encode immediate rewards, and how states predict the next state to come (state flow) is being learnt. This would imply that a model-based reward expectation would have to be calculated before a prediction error can be generated. In practice, it is thought that parallel systems might be computing future expected rewards – a fast and inflexible model-free approach, and a slow and flexible model-based approach located in...
These two systems might compete when predicting expected rewards, with the system that makes the most reliable predictions having the final word.25

Last of all, studies have started to look at the effects of reward risk and uncertainty in guiding behavioral choice.67-69 It has also been proposed that the brain might be explicitly encoding reward risk as a separate signal, so as to arrive at optimal economic decisions.70

**DISCUSSION**

In this thesis I will provide evidence that reward expectations are not computed solely in a model-free approach, as advocated by reinforcement learning, but that explicit, model-based encoding of the structure of the task being solved better explains behavioral choices, as well as reward expectations and prediction errors in the human brain (Chapter 2). I will then look at how different brain regions interact to reach model-based decisions (Chapter 3) and use a whole brain approach to predict what a subject’s next decision will be using single trial fMRI signals. Finally I study how subjects with localized amygdala lesions impact the generation of expected reward signals for the guidance of behavioral choice (Chapter 4). A different, albeit more complex, task was then used to corroborate and extend these results. Subjects participated in a competitive game in which they had to predict the opponent’s next choice to guide their own actions (Chapter 5). This involved not only making a model of the opponent, but players had to understand how their own action influenced the opponent’s behavior.

The tasks used in this thesis to explore model-based decision making in humans incorporate two facets. The first is that environment states are not explicit, and subjects have to create an abstract internal representation of states to solve the tasks. Secondly, the association of reward contingencies with internal states is assumed known (from training before the task or from explicit instructions), and thus this thesis is not a study of learning, but a study of abstract model-based state inference to guide decision making. Optimal state inference can be formulated using Bayesian estimation (Chapter 2), but simpler equivalent dynamic equations are later derived (Chapter 4 and 5). A general introduction to Bayesian inference is provided in Appendix A; and the link between Bayesian inference and the
equivalent dynamic equations is made explicit in Appendix B. Conclusions for each study can be found in the associated chapters.

This body of work shows that humans are not guided solely by model-free RL mechanisms, but that they do incorporate complex knowledge of the task to update the values of all choices accordingly. We hypothesize that this is done by creating an abstract model of environment states on which the task is solved, and from which expected values are then extracted to guide choice – a process known as model-based decision making. Furthermore, fMRI BOLD signals subsequent to a subject’s choice reliably encode the expected reward, as calculated from these model-based algorithms in ventromedial PFC. More generally, this work shows the value of using optimal choice models to explain behavior, and then using the internal model variables to tease apart neural processes in the brain.

Many questions on model-based decision making are left unanswered in this thesis. Most encompassing is the question of how the brain learns the structure of the model, with its internal abstract states and associated expected reward values, on which it later infers state activities to guide decision making as shown in this work. This can be broken in two: the learning and categorization of environment states, and the learning of how each state predicts the state that will follow with the flow of time. Understanding how these two processes are learnt, and the role that reward modulation has, will be an exciting task for further research.
Chapter 2

MODEL-BASED DECISION MAKING IN HUMANS

Many real-life decision making problems incorporate higher-order structure, involving interdependencies between different stimuli, actions, and subsequent rewards. It is not known whether brain regions implicated in decision making, such as ventromedial prefrontal cortex, employ a stored model of the task structure to guide choice (model-based decision making) or merely learn action or state values without assuming higher-order structure, as in standard reinforcement learning. To discriminate between these possibilities we scanned human subjects with fMRI while they performed a simple decision making task with higher-order structure: probabilistic reversal learning. We found that neural activity in a key decision making region – ventromedial prefrontal cortex – was more consistent with a computational model that exploits higher-order structure, than with simple reinforcement learning. These results suggest that brain regions such as ventromedial prefrontal cortex employ an abstract model of task structure to guide behavioral choice, computations that may underlie the human capacity for complex social interactions and abstract strategizing.

INTRODUCTION

Adaptive reward-based decision making in an uncertain environment requires the ability to form predictions of expected future reward associated with particular sets of actions, and then bias action selection toward those actions leading to greater reward\textsuperscript{35, 71}. Reinforcement learning models (RL) provide a strong theoretical account for how this might be implemented in the brain\textsuperscript{15}. However, an important limitation of these models is that they fail to exploit higher-order structure in a decision problem, such as interdependencies between different stimuli, actions, and subsequent rewards. Yet, many real-life decision problems do incorporate such structure\textsuperscript{53, 72, 73}.

To determine whether neural activity in brain areas involved in decision making is accounted for by a computational decision making algorithm incorporating an abstract model of task structure or else by simple reinforcement learning (RL), we conducted a functional Magnetic Resonance Imaging (fMRI) study where subjects performed a simple decision making problem with higher-order structure: probabilistic reversal learning\textsuperscript{53, 74, 75} (Fig. 2.1A). The higher-order structure in this task is the anti-correlation between the reward distributions associated with the two options, and the knowledge that the contingencies will reverse.

To capture the higher-order structure in the task, we used a Markov model (Fig. 2.1B) that incorporates an abstract state variable: the “choice state”. The model observes an outcome (gain; loss) with a probability that depends on the choice state; if the choice state is “correct” then the outcome is more likely to be high; otherwise the outcome is more likely to be low. The observations are used to infer whether the choice state is correct or not. The crucial difference between a simple RL model and the (Markov) model with an abstract, hidden state, is that in the former only the value of the chosen option is updated, whereas the valuation of the option that was not chosen does not change (see Methods); while in the latter, state-based model, both choice expectations change: if stimulus A is chosen and the probability that the choice state is “correct” is estimated to be, say, 3/4, then the probability that the other stimulus, B is correct is assumed to be 1/4 (=1-3/4).
One region that may be especially involved in encoding higher-order task structure is the prefrontal cortex (PFC). This region has long been associated with higher-order cognitive functions, including working memory, planning and decision making\textsuperscript{64-66}. Recent neurophysiological evidence implicates PFC neurons in encoding abstract rules\textsuperscript{76, 77}. On these grounds we predicted that parts of human PFC would correlate better with an abstract state-based decision algorithm than with simple RL. We focused on parts of the PFC known to play an important role in reward-based decision making, specifically its ventral and medial aspects\textsuperscript{65, 74}. 
RESULTS

Behavioral Measures
The decision to switch is implemented on the basis of the posterior probability that the last choice was incorrect. The higher this probability, the more likely a subject is to switch (Fig. 2.1C). The state-based model predicts subjects’ actual choice behavior (whether to switch or not) with an accuracy of 92±2%. On average, subjects made the objectively correct choice (chose the action associated with the high probability of reward) on 61±2% of trials, which is close to the performance of the state-based model (using the parameters estimated from subjects’ behavior), that correctly selected the best available action on 63% of trials. This is also close to the maximum optimal performance of 64%, as measured by a model using the actual task parameters.

Prior correct signal in the Brain
The model estimated prior probability that the current choice is correct (prior correct) informs about the expected reward value of the currently chosen action. The prior correct signal was found to have a highly statistically significant correlation with neural activity in medial prefrontal cortex (mPFC), adjacent orbitofrontal cortex (OFC), and the amygdala bilaterally (Fig. 2.2; the correlation in medial PFC was significant at a corrected level for multiple comparisons across the whole brain at p<0.05). These findings are consistent with previous reports of a role for ventromedial PFC and amygdala in encoding expected reward value55, 78-81. This evidence has, however, generally been interpreted in the context of RL models.

In order to plot activity in medial PFC against the prior probabilities, we sorted trials into one of 5 bins to capture different ranges in the prior probabilities and fitted each bin separately to the fMRI data. This analysis showed a strong linear relation between the magnitude of the evoked fMRI signal in this region and the prior correct probabilities (Fig. 2.2C). We also extracted the % signal change time-courses from the same region and show these in Fig. 2.2D, plotted separately for trials associated with high and low prior probabilities. The time-courses show an increase in signal at the time of the choice
reflected on trials with a high prior correct, and a decrease in signal at the time of the choice for trials with a low prior correct.

**Posterior – prior correct update**

The difference between the posterior correct (at the time of the reward outcome) and the prior correct can be considered an update signal of the prior probabilities once a reward/punishment is received. This signal was significantly correlated with activity in ventromedial PFC as well as in other brain areas such as the ventral striatum (Fig. 2.2B). This update is also reflected in the time-course plots in Fig. 2.2D. Trials with a low prior in which a reward is obtained show an increase in signal at the time of the outcome, whereas trials with a high prior in which a punishment is obtained result in a decrease in signal at outcome. Thus the response at the time of the posterior differs depending on the prior probabilities and whether the outcome is a reward or punishment, fully consistent with the notion that this reflects an update of the prior probabilities.

**Abstract-state model vs. standard RL: The response profile of neural activity in human ventromedial prefrontal cortex**

The prior correct signal from the state-based model is almost identical to the expected reward signal from the RL model. Nevertheless, our paradigm permits sharp discrimination between the two models. The predictions of the two models differ immediately following a switch in subjects’ action choice. According to both models, a switch of stimulus should be more likely to occur when the expected value of the current choice is low, which will happen after receiving monetary losses on previous trials. What distinguishes the two models is what happens to the expected value of the newly chosen stimulus after subjects switch. According to simple RL, the expected value of this new choice should also be low, because that was the value it had when the subject had previously stopped selecting it and switched choice (usually after receiving monetary losses on that stimulus). As simple RL only updates the value of the chosen action, the value of the non-chosen action stays low until the next time that action is selected. However, according to a state-based inference model, as soon as a subject switches action choice, the expected reward value of the newly chosen action should be high, because the known structure of the reversal task incorporates
the fact that once the value of one action is low, the value of the other is high. Thus, in a brain region implementing abstract state-based decision making, the prior correct signal (which reflects expected value) should jump up following reversal, even before an outcome (and subsequent prediction error) is experienced on that new action. In RL, the value of the new action will only be updated following an outcome and subsequent prediction error. This point is illustrated in Fig. 2.3A where the model predicted expected value signals are plotted for simple RL and for the state-based model, before and after reversal. Changes in activation in ventromedial PFC upon choice switches correspond to those predicted by the abstract state-based model: activation decreases after a punishment and if subject does not switch, but increases upon switching, rejecting the RL model in favor of the model with an abstract hidden state (see also Fig. S2.3).

To further validate this point, we conducted an fMRI analysis in which we pitted the state-based model and the best fitting (to behavior) RL algorithm against each other, to test which of these provides a better fit to neural activity. A direct comparison between the regression fits for the state-based model and those for RL revealed that the former was a significantly better fit to the fMRI data in medial PFC at p<0.001 (Fig. 2.3B). While the peak difference was in medial PFC, the state-based model also fit activity better in medial OFC at a slightly lower significance threshold (p<0.01). This suggests that abstract state-based decision making may be especially localized to the ventromedial PFC.

**Prior incorrect**

We also tested for regions that correlated negatively with the prior correct, that is areas correlating positively with the prior probability that the current action is incorrect. This analysis revealed significant effects in other sectors of prefrontal cortex: specifically right dorsolateral prefrontal cortex (rDLPFC), right anterior insular cortex, and anterior cingulate cortex (Fig. 2.4A). Fig. 2.4B shows the relation between the BOLD activity and the model prior incorrect signal in rDLPFC.
Behavioral decision to switch
Finally, we tested for regions involved in implementing the behavioral decision itself (to switch or stay). Enhanced responses were found in anterior cingulate cortex and anterior insula on switch compared to stay trials (Fig. 2.4C). This figure shows that regions activated during the decision to switch are in close proximity to those areas that are significantly correlated with the prior probability that the current choice is incorrect, as provided by the decision model.
DISCUSSION

In this study we set out to determine whether during performance of a simple decision task with a rudimentary higher-order structure, human subjects engage in state-based decision making in which knowledge of the underlying structure of the task is used to guide behavioral decisions, or if, on the contrary, subjects use the individual reward history of each action to guide their decision making without taking into account higher-order structure (standard RL). The decision making task we used incorporates a very simple higher-order structure: the probability that one action is correct (i.e., leading to the most reward) is inversely correlated with the probability that the other action is incorrect (i.e., leading to the least reward). Over time the contingencies switch, and once subjects work out that the current action is incorrect they should switch their choice of action. We have captured state-based decision making in formal terms with an elementary Bayesian Hidden Markov computational model that incorporates the task structure (by encoding the inverse relation between the actions and featuring a known probability that the action reward contingencies will reverse). By performing optimal inference on the basis of this known structure, the model is able to compute the probability that the subject should maintain their current choice of action or switch their choice of action.

The critical distinction between the state-based inference model and standard RL is what happens to the expected value of the newly chosen stimulus after subjects switch. According to standard RL, the expected value of the new choice should be low, because that was the value it had when the subject had previously stopped selecting it (usually after receiving monetary losses on that stimulus). By contrast, the state-based algorithm predicts that the expected value for the newly chosen action should be high, because, unlike standard RL, it incorporates the knowledge that when one action is low in value the other is high. By comparing neural activity in the brain before and after a switch of stimulus, we have been able to show that, consistent with state-based decision making, the expected value signal in ventromedial prefrontal cortex jumps up even before a reward is delivered on the newly chosen action. This updating therefore does not occur at the time of outcome via a standard reward prediction error (as in standard RL). Rather, the updating seems to
occur using prior knowledge of the task structure. This suggests that ventromedial prefrontal cortex participates in state-based inference rather than standard RL.

Our Bayesian Markov model is just one of a family of models which incorporates the simple abstract structure of the task. Thus, while we show that vmPFC implements state-based inference, we remain agnostic about the particular computational process by which this inference is implemented. Furthermore, our findings do not rule out a role for simple RL in human decision making, but rather open the question of how abstract-state based inference and simple RL might interact with each other in order to control behavior\textsuperscript{25}. This also raises the question of whether the dopaminergic system, whose phasic activity has been traditionally linked with a reward prediction error in simple RL, subserves a similar function when the expected rewards are derived from an abstract state representation. An important signal in our state-based model is the posterior correct, which represents an update of the prior correct probability based on the outcome experienced on a particular trial. The difference between the posterior and the prior looks like an error signal, similar to prediction errors in standard RL, except that the updates are based on the abstract states in the model. We found significant correlations with this update signal (posterior-prior) in ventral striatum and mPFC, regions that have previously been associated with prediction error coding in neuroimaging studies\textsuperscript{60, 62, 63, 82, 83} (Fig. 2.2B). These findings are consistent with the possibility that ventromedial prefrontal cortex is involved in encoding the abstract state-space, while standard reinforcement learning is used to learn the values of the abstract states in the model, an approach known as model-based reinforcement learning\textsuperscript{15, 84}.

The final decision whether to switch or stay, was associated with activity in anterior cingulate cortex and anterior insula, consistent with previous reports of a role for these regions in behavioral control\textsuperscript{74, 75, 85-88}. These regions are in close proximity to areas that were significantly correlated with the prior probability that the current choice was incorrect, as provided by the decision model. A plausible interpretation of these findings is that anterior insula and anterior cingulate cortex may actually be involved in using information about the inferred choice probabilities in order to compute the decision itself.
In the present study we provide evidence that neural activity in ventromedial PFC reflects learning based on abstract states that capture interdependencies. Our results imply that the simple RL model is not always appropriate in the analysis of learning in the human brain. The capacity of prefrontal cortex to perform inference on the basis of abstract states shown here could also underlie the ability of humans to predict the behavior of others in complex social transactions and economic games, and accounts more generally for the human ability of abstract strategizing.\(^8\)
MATERIALS AND METHODS

Subjects
Sixteen healthy normal subjects participated in this study (14 right handed; 8 female). The subjects were pre-assessed to exclude those with a prior history of neurological or psychiatric illness. All subjects gave informed consent and the study was approved by the Institute Review Board at Caltech.

Task description
Subjects participated in a simple decision-making task with higher-order structure: probabilistic reversal learning. On each trial they are simultaneously presented with the same two arbitrary fractal stimuli objects (left-right spatial position random) and asked to select one. One stimulus is designated the correct stimulus in that choice of that stimulus leads to a monetary reward (winning 25 cents) on 70% of occasions, and a monetary loss (losing 25 cents) 30% of the time. Consequently choice of this “correct” stimulus leads to accumulating monetary gain. The other stimulus is “incorrect,” in that choice of that stimulus leads to a reward 40% of the time and a punishment 60% of the time, leading to a cumulative monetary loss. The specific reward schedules used here are based on those used in previous studies of probabilistic reversal learning\textsuperscript{53, 90}. After having chosen the correct stimulus on 4 consecutive occasions, the contingencies reverse with a probability of 0.25 on each successive trial. Once reversal occurs, subjects then need to choose the new correct stimulus on 4 consecutive occasions before reversal can occur again (with 0.25 probability). Subjects were informed that reversals occurred at random intervals throughout the experiment but were not informed of the precise details of how reversals were triggered by the computer (so as to avoid subjects using explicit strategies such as counting the number of trials to reversal). The subject’s task is to accumulate as much money as possible, and thus keep track of which stimulus is currently correct and choose it until reversal occurs. In the scanner, visual input was provided with Restech (Resonance Technologies, Northridge, CA, USA) goggles, and subjects used a button box to choose a stimulus. At the same time that the outcome was revealed, the total money won was also displayed. In addition to the reversal trials we also included null event trials, which were
33% of the total number of trials and randomly intermixed with the reversal trials. These trials consist of the presentation of a fixation cross for 7 secs. Before entering the scanner, subjects were informed that they would receive what they earned plus an additional $25 dollars. If they sustained a loss overall then that loss would be subtracted from the $25 dollars. On average, subjects accumulated a total of $3.80 (+/- $0.70) during the experiment.

**Pre-scan training**

Before scanning, the subjects were trained on three different versions of the task. The first is a simple version of the reversal task, in which one of the two fractals yields monetary rewards 100% of the time and the other monetary losses 100% of the time. These then reverse according to the same criteria as in the imaging experiment proper, where a reversal is triggered with probability 0.25 after 4 consecutive choices of the correct stimulus. This training phase is ended after the subject successfully completes 3 sequential reversals. The second training phase consists of the presentation of two stimuli that deliver probabilistic rewards and punishments as in the experiment, but in which the contingencies do not reverse. The training ends after the subject consecutively chooses the “correct” stimulus 10 times in a row. The final training phase consists of the same task parameters as in the actual imaging experiment (stochastic rewards and punishments, and stochastic reversals, as described above). This phase ends after the subject successfully completes 2 sequential reversals. Different fractal stimuli were used in the training session to those used in the scanner. Subjects were informed that they would not receive remuneration for their performance during the training session.

**Reinforcement learning model**

Reinforcement learning (RL) is concerned with learning predictions of the future reward that will be obtained from being in a particular state of the world or performing a particular action. Many different varieties of RL algorithms exist. In this study we used a range of well known RL algorithms to find the one that provided the best fit to subjects’ choice data (see Fig. S2.1 for the comparison of behavioral fits between algorithms). The best fitting RL algorithm was then compared against the state-based decision model for the results
reported in the study. See the Behavioral Data Analysis section for a description of the model-fitting procedure.

The best fitting algorithm to subjects’ choice data was a variant of Q-learning\textsuperscript{91}, in which action values are updated via a simple Rescorla-Wagner (RW) rule\textsuperscript{14}. On a trial \( t \) in which action \( a \) is selected, the value of action \( a \) is updated via a prediction error \( \delta \):

\[
V_a(t + 1) = V_a(t) + \eta \delta(t),
\]  

(2.1)

where \( \eta \) is the learning rate. The prediction error \( \delta(t) \) is calculated by comparing the actual reward received \( r(t) \) after choosing action \( a \) with the expected reward for that action:

\[
\delta(t) = r(t) - V_a(t).
\]  

(2.2)

When choosing between two different states (\( a \) and \( b \)), the model compares the expected values to select which will give it the most reward in the future. The probability of choosing state \( A \) is:

\[
P(A) = \sigma\left( \beta \left( V_a - V_b \right) - \alpha \right),
\]  

(2.3)

where \( \sigma(z) = 1/(1 + \exp(-z)) \) is the Luce choice rule\textsuperscript{18} or logistic sigmoid, \( \alpha \) indicates the indecision point (when it’s equiprobable to make either choice), and \( \beta \) reflects the degree of stochasticity in making the choice (i.e., the exploration/exploitation parameter).

**Abstract state-based model**

We constructed a Bayesian Hidden State Markov Model HMM, see \textsuperscript{92} (HMM) that incorporates the structure of the probabilistic reversal learning task (Fig. 2.1B), and which can be solved optimally with belief propagation techniques\textsuperscript{24}. \( X_t \) represents the abstract hidden state (correct or incorrect choice) that subjects have to infer at time \( t \). \( Y_t \) represents the reward (positive) or punishment (negative) value subjects receive at time \( t \). And \( S_t \) represents whether subjects switched or stayed between time \( t-1 \) and time \( t \). The conditional probabilities linking the random variables are as follows:
\[
P(X_t / X_{t-1}, S_t = stay) = \begin{pmatrix} 1 - \delta & \delta \\ \delta & 1 - \delta \end{pmatrix}
\]

\[
P(X_t / X_{t-1}, S_t = switch) = \begin{pmatrix} \delta & 1 - \delta \\ 1 - \delta & \delta \end{pmatrix}
\]

(2.4)

\[
P(Y_t / X_t = i) = N(\mu_i, \sigma_i).
\]

The first two conditional probabilities describe the transition probabilities of the hidden state variable from trial to trial. If the subjects stay (make the same choice as in the previous trial) and their last choice was correct \((X_{t-1} = \text{correct})\), then their new choice is incorrect \((X_t = \text{incorrect})\) with probability \(\delta\), where delta is the reversal probability (probability that the contingencies in the task reverse) and which was considered to be learnt during training. Likewise, if the subjects stay and their last choice was incorrect \((X_{t-1} = \text{incorrect})\), then their new choice will be correct with probability \(\delta\). On the other hand, if subjects switch, with their last choice being incorrect, the new choice might still be incorrect with probability \(\delta\). The state transition matrices in equation 2.4 incorporate the structural relationship between the reversing task contingencies and subjects’ switches. To complete the model, we include the probability of receiving a reward \(P(Y/X)\) given the state (correct or incorrect choice) the subjects are in. This was modeled as a Gaussian distribution whose mean is the expected monetary reward each state has. In the present task, the actual expected value of the correct choice is 10 cents and the expected value of the incorrect choice is -5 cents. However, to allow for possible variation in the accuracy of subjects’ estimates of the expected values of each choice, we left these expected values as free parameters when fitting the Bayesian model to each subject’s behavioral data. Fitted parameters for the reversal probability and expected rewards were close to the actual experimental parameters (Table S2.1).

With \(P(X_0) = (0.5, 0.5)\) at the beginning of the experiment, Bayesian inference was carried out to calculate the posterior probability of the random variable \(X\) (correct/incorrect choice) given the obtained rewards and punishments (variable \(Y\)) and the subjects’ switches.
(variable $S$) using causal belief propagation (equations 2.5-2.6). Equation 2.5 specifies the subjects’ prior, or belief that they will be at a given internal state at time $t$ as a consequence of their choice $S_t$ and the internal state posterior from the previous trial. Equation 2.6 updates this prior with the observed reward/punishment $Y_t$ to obtain the current posterior, or optimal assessment of the state at time $t$. These equations have the Markov property that knowledge of only the posterior from the previous trial, as well as the last reward/punishment and behavioral action are needed to calculate the posterior of the next trial. An introduction to HMMs is provided in the supplementary methods section at the end of this chapter, as well as in Appendix A.

$$
\text{Prior} (X_t = \text{correct}) = \sum_{X_{t-1} \text{states}} P(X_t = \text{correct} / X_{t-1}, S_t) \text{Posterior} (X_{t-1}) 
$$

(2.5) $$
\text{Posterior} (X_t = \text{correct}) = \frac{P(Y_t / X_t = \text{correct}) \text{Prior} (X_t = \text{correct})}{\sum_{X_t \text{states}} P(Y_t / X_t) \text{Prior} (X_t)}
$$

(2.6)

For the reversal task, the consequence of action switch (or stay) is linear with the inferred posterior probability that the subjects are making the incorrect (or correct) choice (and so are the expected rewards). The decision to switch is thus based on the probability that the current choice is incorrect $\text{Posterior}(X_t = \text{incorrect})$ (the close correspondence between the model-estimated posterior that the current choice was incorrect and subjects’ actual choice behavior is illustrated in Fig. 2.1C). We assume a stochastic relation between actual choice and the probability that the current choice is incorrect, and use the logistic sigmoid as in equation 2.3:

$$
P(\text{switch}) = \sigma(\beta(P_{\text{incorrect}} - \alpha)).$$

(2.7)

The state-based model we use here assumes that subjects use a fixed probability of reversal which is uniform on all trials in which the correct stimulus is being chosen. However, in actuality the probability of reversal is not uniformly distributed over all the trials, because after subjects switch their choice the reversal probability is set to zero until subjects make the correct choice on four consecutive occasions. We compared a version of the state-based
model which incorporates the full reversal rule (0 probability of reversal until four consecutive correct choices are made, and a fixed probability thereafter) to that which incorporates a simple rule based on a single fixed probability. The latter model was found to provide a marginally better fit (with a log likelihood of -0.29 compared to -0.40 for the full model) to subjects actual behavioral choices. This justifies use of a state-based model with this simplified reversal rule in all subsequent analyses.

**Behavioral Data Analysis**

Both the RL and state-based model decision probabilities \( P(\text{switch/stay}) \) were fitted against the behavioral data \( B(\text{switch/stay}) \). The state-based model calculates the probability of switching through equation 2.7. The RL model computes the probability of choosing one stimulus vs. another, but can be converted to a switch/stay probability based on the subject’s previous selection (i.e., \( P(\text{switch})=P(\text{choose A}) \) if the subject chose \( B \) in the previous trial, and vice-versa). On average, subjects switched 22±2 times during the experiment, out of around 104 trials, so we used a maximum log likelihood fitting criteria that weighed switching and staying conditions equally:

\[
\log L = \frac{\sum_{\text{switch}} B_{\text{switch}} \log P_{\text{switch}}}{N_{\text{switch}}} + \frac{\sum_{\text{stay}} B_{\text{stay}} \log P_{\text{stay}}}{N_{\text{stay}}}. \tag{2.8}
\]

Model parameters were fitted using a variant of a simulating annealing procedure. A comparison of the log likelihoods of the Bayesian model, and a number of RL models is shown in Fig. S2.1, and a time plot of subject choices vs. model predictions in Fig. S2.2. The Bayesian model has a better log likelihood fit than the best-fitting RL model (\( P < 10^{-7} \), paired t-test). This is also true even when using a penalized log likelihood measure (Bayes Information Criterion – BIC) that takes into account the number of free parameters in each model, shown in equation 2.9; where \( M \) is the number of free parameters (5 for the Bayesian model, 3 for the model-free RL) and \( N \) the total number of data points.

\[
\text{BIC} = -2 \log L + M \frac{\log N}{N} \tag{2.9}
\]
The mean fitted parameter values across subjects for the Bayesian model and the best-fitting RL model are shown in Table S2.1. These parameters were used when fitting the models to the fMRI data. We assumed subjects learned the task structure and reward contingencies during the training period, and then kept these parameters fixed during task execution.

We note that while the approach we use here of deriving best-fitting parameters from subjects’ behavior and then regressing the model with these parameters against the fMRI data is perhaps the most parsimonious way to constrain our model-based analysis, this approach assumes that behavior is being controlled by a single unitary learning system with a single set of model parameters. However, it is possible that behavior may be controlled by multiple parallel learning systems, each with distinct model parameters, and, as such, these multiple learning systems would not be discriminated using our approach.

**fMRI data acquisition**

Functional imaging was conducted using a Siemens 3.0 Tesla Trio MRI scanner to acquire gradient echo T2* weighted echo-planar images (EPI). To optimize functional sensitivity in OFC we acquired the data using an oblique orientation of 30° to the AC-PC line. A total of 580 volumes (19 minutes) were collected during the experiment in an interleaved-ascending manner. The imaging parameters were: echo time, 30ms; field-of-view, 192mm; in-plane resolution and slice thickness, 3mm; TR, 2 seconds. High-resolution T1-weighted structural scans (1x1x1mm) were acquired for anatomical localization. Image analysis was performed using SPM2 (Wellcome Department of Imaging Neuroscience, Institute of Neurology, London, UK). Pre-processing included slice timing correction (centered at TR/2), motion correction, spatial normalization to a standard T2* template with a resampled voxel size of 3mm, and spatial smoothing using an 8mm gaussian kernel. Intensity normalization and high-pass temporal filtering (128 secs) were also applied to the data.
fMRI data analysis

The event-related fMRI data was analyzed by constructing sets of delta (stick) functions at the time of the choice, and at the time of the outcome. Additional regressors were constructed by using the model estimated prior probabilities as a modulating parameter at the time of choice; and the state-based prediction error signal (posterior-prior probabilities) as a modulating parameter at the time of outcome. In addition, we modeled subjects’ behavioral decision (switch vs. stay) by time-locking a regressor to the expected time of onset of the next trial (two seconds after the outcome is revealed). All of these regressors were convolved with a canonical hemodynamic response function (hrf). In addition, the 6 scan-to-scan motion parameters produced during realignment were included to account for residual motion effects. These were fitted to each subject individually, and the regression parameters were then taken to the random effects level to obtain the results shown in Figs. 2.2 and 2.4. All reported fMRI statistics and p-values arise from group random effects analyses. We report those activations as significant in a priori regions of interest that exceed a threshold of p<0.001 uncorrected, whereas activations outside regions of interest are reported only if they exceed a threshold of p<0.05 following whole brain correction for multiple comparisons. Our a priori regions of interest are: prefrontal cortex (ventral and dorsal aspects), anterior cingulate cortex, anterior insula, amygdala, and striatum (dorsal and ventral), as these areas have previously been implicated in reversal learning and other reward-based decision making tasks e.g. 74, 82.

Time series of fMRI activity in regions of interest (shown in Fig. 2.2D) were obtained by extracting the first eigenvariate of the filtered raw time-series (after high-pass filtering and removal of the effects of residual subject motion) from a 3mm sphere centered at the peak voxel (from the random effects group level). This was done separately for each individual subject, binned according to different trial types and averaged across trials and subjects. SPM normalizes the average fMRI activity to 100, so that the filtered signal is considered as a percentage change from baseline. It is to be noted that the time-series are not generated using canonical hrf functions. More specifically, peak BOLD activity is lagged with respect to the time of the event that generated it. For example, activity arising as a consequence of
neural activity at the time of choice will have its maximum effect 4-6 seconds after the time of choice, as expressed in the time-series plot.

We also compared the best fitting model-free RL and Bayesian algorithms directly (Fig. 2.3B) by fitting both models at the same time with the fMRI data. To make both models as similar as possible, we used the normalized value and prediction error signals from the Rescorla Wagner model as regressors (modulating activity at the time of the trial onset and outcome respectively), and the normalized prior correct and prediction error (posterior-prior correct) from the state-based model as regressors (modulating activity at the time of the trial onset and outcome, respectively). Separate Reward and Punishment regressors were also fitted at the time of the outcome. Prior correct and value contrasts were calculated at the individual level and then taken to the random effects level to determine which areas had a better correlation with the state-based model.

In order to calculate the predicted value and prior correct signals for the standard RL and state-based model shown in Fig. 2.3, we calculated the expected value (from the RW model) and prior correct value (derived from the state-based model) on all trials in which subjects received a punishment and for the immediately subsequent trial. We then sorted these estimates into two separate categories according to whether subjects switched their choice of stimulus or maintained their choice of stimulus (stay) on the subsequent trial.
A
- Trial Onset
- 500ms
- Subject choice
- 2s choice disappears
- 5s Reward delivered
- 6s Reward disappears
- 7s Trial ends

B
- $X_{t-1}$
- $Y_{t-1}$
- $X_t$
- $Y_t$

C
- Incorrect choice posterior
- Action probability
  - Stay
  - Switch

Graphs showing action probability and incorrect choice posterior with bars for stay and switch actions.
Figure 2.1. Reversal task setup and state-based decision model. (A) Subjects choose one of two fractals, which on each trial are randomly placed to the left or right of the fixation cross. Once a stimulus is selected by the subject it increases in brightness, and remains on the screen until 2 seconds after the choice. After a further 3s, a reward (winning 25 cents, depicted by a quarter dollar coin) or punishment (losing 25 cents, depicted by a quarter dollar coin covered by a red cross) is delivered, with the total money earned displayed at the top. One stimulus is designated the correct stimulus in that choice of that stimulus leads to a monetary reward on 70% of occasions, and a monetary loss 30% of the time. Consequently choice of this ‘correct’ stimulus leads to accumulating monetary gain. The other stimulus is ‘incorrect’, in that choice of that stimulus leads to a reward 40% of the time and a punishment 60% of the time, leading to a cumulative monetary loss. After subjects choose the correct stimulus on 4 consecutive occasions, the contingencies reverse with a probability of 0.25 on each successive trial. Subjects have to infer the reversal took place and switch their choice, at which point the process is repeated. (B) We constructed an abstract state-based model that incorporates the structure of the reversal task in the form of a Bayesian Hidden State Markov Model, which uses previous choice and reward history in order to infer the probability of being in the correct/incorrect choice state. The choice state changes (“transits”) from one period to another depending on (i) the exogenously given chance that the options are reversed (the good action becomes the bad one and v.v.), and (ii) the control (if subject switches when the actual – but hidden – choice state is “correct” then the choice state becomes “incorrect” and v.v.). In the diagram, $Y$ is the observed reward/punishment, $S$ the observed switch/stay action, and $X$ the abstract correct/incorrect choice state that is inferred at each time step (see Methods). Arrows indicate the causal relations among random variables. (C) Observed choice frequencies that subjects switch (black) or stay (grey) against the state-based model’s inferred posterior probability that their last choice was incorrect. The higher the posterior incorrect probability, the more likely subjects switch (relative choice frequencies are calculated separately for each posterior probability bin).
**Figure 2.2. Correct choice prior, and posterior - prior update signals in the brain.**

(A) Brain regions showing a significant correlation with the prior correct signal from the state-based decision model (time-locked to the time of choice). Strong correlations with prior correct were found in ventromedial prefrontal cortex (mPFC: 6, 57, -6mm; $z=5.33$; OFC: 0, 33, -24mm; $z=4.04$), as well as in posterior dorsal amygdala (extending into anterior hippocampus). The activations are shown superimposed on a subject averaged structural scan and the threshold is set at $p<0.001$.

(B) Brain regions correlating with the posterior-prior update signal. This is a form of prediction error signal that reflects the difference in value between the prior probability that the choice will be correct and the posterior probability that the choice was correct following receipt of the outcome (a reward or punishment). This signal is significantly correlated with activity in the bilateral ventral striatum (-24,3,-9mm; $z=4.64$ and 18,3,-15mm; $z=4.48$) and medial PFC (-6,54,-24 mm; $z=3.54$). These fMRI contrasts are from group random effects analyses.

(C) The relationship between fMRI responses in medial prefrontal cortex (yellow circle in panel a) at the time of choice and the prior correct signal from the state-based model showed a strong co-linearity, supporting the idea of an optimal inference of state probabilities. In order to plot this activity against the prior probabilities, we sorted trials into one of 5 bins to capture different ranges in the prior probabilities, and fitted each bin separately to the fMRI data.

(D) The time course for the averaged % signal change in this same region (mPFC) is shown separately for trials with a high prior correct signal (prob. > 0.65) and low prior correct signal (prob. < 0.5). Error bars depict the standard error of the mean across all trials of that type. Trials are also separated according to whether a reward or a punishment was received at the time of outcome to illustrate updating of the signal following feedback. The leftmost shaded area indicates the period (1 sec in length) in which subjects made their choice, and the second shaded area the period in which subjects were presented with their rewarding or punishing feedback. Trials with a low prior in which a reward is obtained show an increase in signal at the time of the outcome (with the peak BOLD activity lagged by 4-6 seconds, see Methods), whereas trials with a high prior in which a punishment is obtained result in a decrease in signal at outcome, consistent with the possibility that the response at the time of outcome reflects an update signal.
A

**Standard RL**

- Value
- Prior correct
- % Bold response

**State-based Model**

**mPFC**

B

**State-based > standard RL**

![Brain Image](image.png)
Figure 2.3. Standard RL and abstract state-based decision models make qualitatively different predictions about the brain activity after subjects switch their choice. (A) Both models predict that if a decision is made to stay after being punished, the next action will have a lower expected value in the next trial (blue line). However, if a decision is made to switch choice of stimulus after being punished, simple RL predicts that the expected value of the new choice will also be low (red line – leftmost panel) because its value was not updated since the last time it was chosen. On the other hand a state-based decision model predicts that the expected value of the new choice will be high, because if the subjects have determined that their choice till now was incorrect (prompted by the last punishment), then their new choice after switching is now correct and has a high expected value (red line – middle panel). Mean fMRI signal changes (time-locked to the time of choice) in medial PFC (derived from a 3mm sphere centered at the peak voxel) plotted before and after reversal (rightmost panel) show that activity in this region is more consistent with the predictions of state-based decision making than that of standard RL, indicating that the expected reward signal in mPFC incorporates the structure of the reversal task. (B) Direct comparison of brain regions correlating with the prior correct signal from the state-based model compared to the equivalent value signal (of the current choice) from the simple RL model. A contrast between the models revealed that the state-based decision model accounts significantly better for neural activity in medial PFC (shown left panel; 6,45,-9mm; z=3.68).
A Prior incorrect

B  

C Switch - stay

B

Regression coefficient

Prior Incorrect

0.3 0.4 0.5 0.6

2.5

1.5

0.5
Figure 2.4. Incorrect choice prior and switch - stay signals in the brain. (A) Brain regions showing a significant correlation with the prior incorrect signal from the state-based algorithm (time-locked to the time of choice). Significant effects were found in right dorsolateral PFC (39, 36, 33mm; z=4.30), anterior cingulate cortex (6, 21, 45mm; z=3.37), and right anterior insula (48, 15, 9mm; z=3.96). The threshold is set at p<0.001. (B) Plot showing relationship between fMRI responses in dorsolateral prefrontal cortex at the time of choice and the prior incorrect signal from the Bayesian model, illustrating strong co-linearity between this signal and activity in this region. (C) Brain regions responding on trials in which subjects decide to switch compared to when they do not switch their choice of stimulus. Significant effects were found in anterior cingulate cortex (-3, 24, 30mm; z=4.54) and anterior insula bilaterally (-39, 18, -12mm; z=4.26; 51, 21, 3mm; z=4.23). The fact that anterior cingulate and anterior insula respond on these switch trials, as well as responding to the prior incorrect signals, suggests that the decision to switch may be implemented in these regions.
SUPPLEMENTARY METHODS

Reinforcement Learning Models

Q-Learning

These algorithms learn what actions to take when in a given state by learning a value of the reward that is expected after taking that action. The simplest form, as depicted in the Methods section in the chapter, updates the value of the action via a simple Rescorla-Wagner (RW) rule\textsuperscript{14}. On a trial $t$ in which action $a$ is selected, the value of action $a$ is updated via a prediction error $\delta$:

$$V_a(t + 1) = V_a(t) + \eta \delta(t), \quad \text{(s2.1)}$$

where $\eta$ is the learning rate. The prediction error $\delta(t)$ is calculated by comparing the actual reward received $r(t)$ after choosing action $a$ with the expected reward for that action:

$$\delta(t) = r(t) - V_a(t) \quad \text{(s2.2)}$$

Thus, action values reflect the immediate subsequent reward that is expected after taking that action. This can be extended so as to learn the cumulative rewards expected in the future, as a consequence of taking a given action. In general, an exponentially discounted measure of future expected reward is used, in which more weight is given to rewards expected in the near future in comparison to rewards expected in the far future:

$$E(R) = \sum_{t=1}^{\infty} \gamma^t E(r_t), \quad 0 < \gamma < 1 \quad \text{(s2.3)}$$

where $\gamma$ is the discount factor that indicates how near and far future rewards are weighed. Q-learning can be extended to learn future discounted expected rewards via Temporal Difference Learning (TD), in which the prediction error used in the value update is:

$$\delta(t) = r(t) + \gamma V_a(t + 1) - V_a(t) \quad \text{(s2.4)}$$
The effective reward obtained in the next time step due to action $a$, is a sum of the immediate reward $r(t)$ and the discounted expected reward due to action $a'$ in the next time step. When using Q-learning with temporal difference prediction errors, $Q(td)$, we fitted a variable number of intermediate states in between the time of choice ($t=1$) and the time of reward outcome ($t=T$), as used in O’Doherty et al.as used in 62. Both Q-learning algorithms, $Q(rw)$ and $Q(td)$, then stochastically choose the action with most value, as explained in Methods (equation 2.3).

**Actor-Critic**

Instead of learning action values directly, as in Q-learning, an alternative approach is to separate value learning and action selection into two stages97. The first stage, or “Critic,” involves policy evaluation, in which the expected future rewards that follow from being in a particular state (corresponding to the average of the expected rewards available for all selected actions in that state) are learnt. The second stage, or “Actor,” uses the prediction error signal ($\delta(t)$) derived from the critic to modify of the probability of choosing a given action so as to increase the average expected reward of the whole policy (equation s2.5).

\[
\begin{align*}
    m_a(t + 1) &= m_a(t) + \eta[\delta_{ab} - P_b]\delta(t) \\
    P_b &= \frac{e^{\beta m_b}}{\sum_i e^{\beta m_i}}
\end{align*}
\]  

The AC(rw) algorithm tested in this chapter uses a Rescorla-Wagner rule (equation s2.2) to calculate the prediction error $\delta(t)$, and the AC(td) algorithm uses Temporal Differences (equation s2.4) with intermediate state steps between the time of choice and the time of reward outcome to calculate the prediction error $\delta(t)$. 
**Advantage Learning**

Advantage Learning is an extension of the actor-critic algorithm, see also. Although advantage learning differs in a number of respects from actor-critic, the main difference in terms of the behavioral fitting of the two models in the present case is that in advantage learning the value of the initial state (time of choice, \( t=1 \)) is not directly estimated but instead is set according to:

\[
V(t = 1) = P_a V_a(t = 2) + P_b V_b(t = 2).
\]  \hspace{1cm} (s2.6)

**Bayesian Hidden State Markov Models**

The key to Bayesian inference is an effective rule, Bayes’ rule, which allows one to infer the probability of a hidden cause from observable variables.

Imagine a medical situation. A patient visits you repeatedly. Upon visit \( t \), you prescribe her medication \( S_t \). The result is a set of symptoms \( Y_t \).

In simple reinforcement learning, you learn the relationship between the medication \( S_t \) and subsequent symptoms \( Y_t \). When this patient returns for visit \( t+1 \), you use the direct relationship between both variables which you learnt from all the past pairs of observables \((S_t, Y_t)\), to determine the choice of medication \( S_{t+1} \). Reinforcement learning is flexible enough to allow for changes in this direct relationship through learning. Effectively,
reinforcement learning discounts observable pairs from earlier visits more than from more recent visits.

Reinforcement learning is only concerned with observables. Bayesian inference, in contrast, allows for the possibility that there is a hidden cause $X_t$ that determines the effectiveness of the medication, i.e., the relationship between the medication $S_t$ and the symptoms $Y_t$. Bayes’ rule allows one to infer what $X_t$ is given the past observable pairs and knowledge on how the hidden causes $X_t$ generate the observable symptoms $Y_t$.

To make the example more concrete, one can imagine that $X_t$ takes on 4 values: $X_t=0$ corresponds to no illness; $X_t=1$ corresponds to a viral infection; $X_t=2$ denotes bacterial infection; $X_t=3$ stands for allergic reaction. $Y_t$ is a measure of nasal symptoms related to colds, flu and allergic reactions. And $S_t$ stands for medication ($S_t=0$: none; $S_t=1$: anti-viral; $S_t=2$: antibiotic; $S_t=3$: antihistamine).

Bayesian inference allows one to model explicitly how the “cause” $X_t$ changes over time – including as a result of the choice variable $S_t$ (medication, in the case of the example). It is often assumed that such changes are Markov, which means that the change in $X$ from visit $t$ to $t+1$ is influenced only by the immediate past ($X_t$).
The interpretation of the various components of this Bayesian hidden state Markov model in the chapter is as follows: $S_t$ denotes the choice of the subject ($S_t=0$ means “stay”; $S_t=1$ corresponds to “switch”); $X_t$ denotes the correctness of the choice ($X_t=0$ means “incorrect choice”; $X_t=1$ means “correct choice”); $Y_t$ is the observed reward ($Y_t$ is a continuous monetary value, rewards being positive and punishments negative).

Further information on Bayes’ rule, Bayesian inference and Bayesian hidden state Markov models can be found by D. MacKay\textsuperscript{100}, M. Jordan\textsuperscript{24}, and Z. Ghahramani\textsuperscript{101}.
Figure S2.1. Comparison of the behavioral fits of the state-based decision model to a variety of standard Reinforcement Learning algorithms, showing that the state-based model provides a better fit to subjects’ behavioral data than a range of RL algorithms. The RL algorithms fitted to the subjects’ behavioral data were: Q-learning Q(rw) using a Rescorla-Wagner update rule, Q-learning Q(td) with intermediate time steps (assumes multiple time steps within a trial and calculates the expected future reward for each time step within a trial\textsuperscript{62, 97}), actor-critic AC(rw) with a Rescorla-Wagner update rule, actor-critic AC(td) with intermediate time steps, and advantage learning (Adv.) – an extension of the actor-critic algorithm\textsuperscript{98}. (A) The log likelihoods of the action predictions (switch vs. stay) of each model show that the state-based model provides the best fit to the data. The second best fitting model is Q-learning using a Rescorla Wagner update rule, which is the RL algorithm we compared to the state-based model in the fMRI analyses. RL models are depicted in light blue. (B) The state-based model shows a better fit to the data even when using the Bayes Information Criterion (BIC) to account for the fact that the state-based model has more free parameters than the RL models (the number of free parameters was 5 for the state-based model, 3 for the Q-learning Q(rw) model, 5 for the Q-learning Q(td) model, 2 for the actor-critic AC(rw), 4 for the actor-critic AC(td), and 4 for Advantage learning).
Figure S2.2. Behavioral data and model predictions for three randomly chosen subjects (subjects 1, 7, 13). The predictions of the state-based decision model (blue line) as to when to switch correspond more closely to the subjects’ actual switching behavior (red bar = switch) as compared to the predictions of the best fitting standard RL algorithm (green line). On top of each graph is the history of received rewards (blue) and punishments (red), with subjects usually switching after a string of punishments.
Figure S2.3. Plot of the model-predicted choice probabilities derived from the best-fitting RL algorithm before and after subjects switch their choice. One possible alternative explanation for the difference in predictions of the abstract state-based model and the RL model, is that in the latter we are showing the predictions of value rather than choice probability. In many RL variants, such as the actor-critic, an anti-correlation between actions is built in when computing the choice probabilities. Thus, it could be argued that, had we plotted choice probability from an RL model instead of value, the predictions of the two models would be much more similar. Here we plot the choice probability data from the best-fitting RL model, which incorporates a form of anti-correlation between actions. In spite of this, we still see that the predictions of the choice probabilities from the RL model do not show the pattern of results we observe for the abstract-state-based model (where the correct choice probability jumps up following reversal). This illustrates that a normalized choice probability signal from standard RL does not emulate the effect predicted by the state-based model, as found to be the case in ventromedial prefrontal cortex.
Table S2.1. Mean parameters across subjects for the behavioral models

<table>
<thead>
<tr>
<th>Q-learning (Rescorla-Wagner)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning Rate $\eta$</td>
</tr>
<tr>
<td>0.84 ± 0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State-based decision model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct mean $\mu_C$</td>
</tr>
<tr>
<td>11.3 ± 1.0 cents</td>
</tr>
</tbody>
</table>
Table S2.2. fMRI activity localization

**a) Prior correct activity**

<table>
<thead>
<tr>
<th>brain region</th>
<th>laterality</th>
<th>x</th>
<th>y</th>
<th>Z</th>
<th>Z-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ventral medial PFC</td>
<td>L/R</td>
<td>6</td>
<td>57</td>
<td>-6</td>
<td>5.33</td>
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<tr>
<td>Posterior amygdala/anterior hippocampus</td>
<td>R</td>
<td>33</td>
<td>-15</td>
<td>-18</td>
<td>4.68</td>
</tr>
<tr>
<td>Dorsomedial PFC (frontopolar gyrus)</td>
<td>L</td>
<td>-9</td>
<td>66</td>
<td>21</td>
<td>4.47</td>
</tr>
<tr>
<td>Medial OFC/Subgenual cingulate cortex</td>
<td>L/R</td>
<td>0</td>
<td>39</td>
<td>-6</td>
<td>4.17</td>
</tr>
<tr>
<td>Medial OFC</td>
<td>L/R</td>
<td>0</td>
<td>33</td>
<td>-24</td>
<td>4.04</td>
</tr>
<tr>
<td>Posterior cingulate cortex</td>
<td>L/R</td>
<td>0</td>
<td>-9</td>
<td>-57</td>
<td>3.98</td>
</tr>
</tbody>
</table>

**b) Posterior correct – Prior correct activity**

<table>
<thead>
<tr>
<th>brain region</th>
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<th>x</th>
<th>y</th>
<th>z</th>
<th>Z-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medial PFC</td>
<td>L</td>
<td>-6</td>
<td>54</td>
<td>24</td>
<td>3.54</td>
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<tr>
<td>Ventral striatum</td>
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<td>-24</td>
<td>3</td>
<td>-9</td>
<td>4.64</td>
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<tr>
<td>Ventral striatum</td>
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<td>3</td>
<td>-15</td>
<td>4.48</td>
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</table>

**c) Prior incorrect activity**

<table>
<thead>
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<th>x</th>
<th>y</th>
<th>z</th>
<th>Z-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dorsolateral PFC</td>
<td>R</td>
<td>39</td>
<td>36</td>
<td>33</td>
<td>4.30</td>
</tr>
<tr>
<td>Anterior insula/frontal operculum</td>
<td>R</td>
<td>48</td>
<td>15</td>
<td>9</td>
<td>3.96</td>
</tr>
<tr>
<td>Anterior cingulate cortex</td>
<td>R</td>
<td>6</td>
<td>21</td>
<td>45</td>
<td>3.37</td>
</tr>
</tbody>
</table>

**d) Switch-Stay activity**

<table>
<thead>
<tr>
<th>brain region</th>
<th>laterality</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>Z-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anterior cingulate cortex</td>
<td>L/R</td>
<td>-3</td>
<td>24</td>
<td>30</td>
<td>4.54</td>
</tr>
<tr>
<td>Anterior insula</td>
<td>R</td>
<td>51</td>
<td>21</td>
<td>3</td>
<td>4.23</td>
</tr>
<tr>
<td>Anterior insula</td>
<td>L</td>
<td>-39</td>
<td>18</td>
<td>-12</td>
<td>4.26</td>
</tr>
</tbody>
</table>
While previous studies have implicated a diverse set of brain regions in reward-related decision making, it is not yet known which of these regions contain information that directly reflects a decision. Here we measured brain activity using fMRI in a group of subjects while they performed a simple reward-based decision making task: probabilistic reversal-learning. We recorded brain activity from 9 distinct regions of interest previously implicated in decision making, and separated out local spatially distributed signals in each region from global differences in signal. Using a multivariate analysis approach, we determined the extent to which global and local signals could be used to decode subjects’ subsequent behavioral choice, based on their brain activity on the preceding trial. We found that subjects’ decisions could be decoded to a high level of accuracy on the basis of both local and global signals even before they were required to make a choice, and even before they knew which physical action would be required. Furthermore, the combined signals from three specific brain areas: anterior cingulate cortex, medial prefrontal cortex, and ventral striatum, were found to provide all the information sufficient to decode subjects’ decisions out of all the regions we studied. These findings implicate a specific network of regions in encoding information relevant to subsequent behavioral choice.

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**INTRODUCTION**

Decision making is a neural process that intervenes between the processing of a stimulus input and the generation of an appropriate motor output. Motor responses are often performed in order to obtain reward, and the obligation of a decision making mechanism is to ensure that appropriate responses are selected in order to maximize available reward. While the neural systems involved in this process have been the subject of much recent research, studies have yet to isolate the specific neural circuits responsible for this decision process. Neural signals have been found that relate to but do not directly reflect this process, such as those pertaining to the expected value or utility of the available actions\(^{35,73}\), responses signaling errors in those predictions\(^{59}\), encoding the value of outcomes received\(^{53}\), as well as responses related to monitoring or evaluation of a previously executed action\(^{87,102,103}\). Such signals have been found in diverse regions throughout the brain, including, anterior cingulate cortex (ACC), medial prefrontal cortex (mPFC), orbitofrontal cortex (OFC), dorsolateral prefrontal cortex (DLPFC), amygdala, and striatum. While complex behavioral decisions are likely to depend on information computed in a widely distributed network, it is not yet known where among this network of brain regions neural activity directly reflects the subsequent behavioral decision as to which action to select.

In order to determine the brain regions where neural activity is directly related to a final behavioral decision, we applied multivariate decoding techniques to our fMRI data. This approach combines the temporal and spatial resolution of event-related fMRI with statistical learning techniques in order to decode on a trial-by-trial basis subjects’ behavior or subjective states directly from their neural activity. Up to now this technique has been used in visual perception, to decode perceptual states and/or perceptual decisions from fMRI signals recorded mainly (though not exclusively) in visual cortical areas\(^{104-108}\). These previous studies have used locally distributed variations in activity to decode visual percepts, under situations where the global mean signals in a given region may show no significant differences between conditions. In the present case, many of our target regions of interest have been found to show global signal changes related to behavioral choice, that
is, large spatially extended cluster areas of activation have previously been reported in these areas in previous fMRI studies\textsuperscript{74, 86}. Here, in addition to testing for global signals, we also tested for the presence of locally distributed signals relevant to behavioral decision making in each of our areas of interest. For this we separated out global and local signals within each region, and explored the separate contributions of signals at these two different spatial scales. We then extend this technique to the multi-region level to determine the contribution of interactions between brain areas in reward-related decision making.

To address this, subjects performed a probabilistic reversal task\textsuperscript{53, 75} while being scanned with fMRI. On each trial, subjects are presented with two fractal stimuli and asked to select one (Fig. 3.1A), with the objective of accumulating as much money as possible. After making a choice, subjects receive either a monetary gain or a monetary loss. However, one choice is “correct,” in that choice of that stimulus leads to a greater probability of winning money and hence to an accumulating monetary gain, while the other choice is “incorrect,” in that choice of that stimulus leads to a greater probability of losing money and hence to an accumulating monetary loss. After a time the contingencies reverse so that what was the correct choice becomes the incorrect choice and vice versa. In order to choose optimally, subjects need to work out which stimulus is correct, and continue to choose that stimulus until they determine the contingencies have reversed, in which case they should switch their choice of stimulus. The goal of our study is to decode subjects’ behavioral choices on a subsequent trial on the basis of neural activity on the preceding trial.

An important feature of this task is that its probabilistic nature precludes subjects from inferring which stimulus is correct on the basis of the outcome received on the previous trial alone, because both correct and incorrect stimulus choices are associated with rewarding and punishing feedback. Rather, subjects need to take into account the history of outcomes received in order to make decisions about what choices to make in future. Furthermore, the two stimuli are presented at random on the left or right of the screen. Thus, on the previous trial, subjects do not know in advance which of two possible motor responses are needed to implement a particular decision until such time as the next trial is triggered. Consequently, our fMRI signal cannot be driven merely by trivial (i.e., non-
decision-related) neural activity pertaining to preparation of a specific motor response (choose left vs. choose right), as such signals are not present before the stimuli are shown. Therefore, the only signals in the brain relevant to decoding choice are those pertaining to the subjects’ abstract decision of whether to maintain their current choice of stimulus or switch their choice to the alternative stimulus, or else the consequences of that decision (e.g., to implement a switch in behavioral responses).

Nine regions of interest were specified a priori (Fig. S3.1), based on previous literature implicating these regions in reward-related decision making. These include the medial and lateral OFC, and adjacent mPFC. These regions have been shown to encode expected reward values, as well as the reward value of outcomes\(^53,\,79,\,109\). Moreover, signals in these regions have been found to relate to behavioral choice – whereby activity increases in medial PFC on trials when subjects maintain their current choices on subsequent trials, compared to when they switch\(^74\).

Another region that we hypothesized might contain signals relevant to behavioral choice, is the ACC. This area is engaged when subjects switch their choice of stimulus on reversal learning tasks\(^74,\,86\), suggesting that signals there relate to behavioral choice. A general role for this region in monitoring action-outcome associations has recently been proposed\(^88\). The region has also been argued to mediate action selection under situations involving conflict between competing responses\(^87\), and action selection between responses with different reward contingencies\(^46\). ACC has also been suggested to play a role in monitoring errors in behavioral responding, or even in decoding when these errors might occur\(^110\). What all of these accounts of anterior cingulate function have in common is that they posit an intervening role for this area between the processing of a stimulus input and the generation of an appropriate behavioral response, even though such accounts differ as to precisely how this region contributes at this intermediate stage. On these grounds we hypothesized that neural signals in ACC would be relevant for decoding subsequent behavioral choices.
Other regions we deemed relevant to decision making include the insular cortex, which has been shown to respond during uncertainty in action choice, as well as under situations involving risk or ambiguity\textsuperscript{67, 68, 111}. Kuhnen and Knutson\textsuperscript{69} showed that neural activity in this region on a previous trial correlated with whether subjects will make a risk-seeking or risk-averse choice in a risky decision making paradigm. We also include in our regions of interest ventral striatum, where activity is linked to errors in prediction of future reward, and dorsal striatum, which is argued to mediate stimulus-response learning as well as goal-directed action selection\textsuperscript{62, 82, 112, 113}. Another region we included is the amygdala, which has been implicated in learning of stimulus-reward or stimulus-punisher associations\textsuperscript{55, 78, 114}.

We analyze the contribution each region of interest gives to the decoding of choice behavior in two ways. In the first, we study each region individually and compare their discriminative power for decoding behavioral choice. This is done by separating fMRI signals in each region into spatially local and spatially global signals, thus disambiguating results that correspond to classic fMRI approaches (global signals), with results that can only be obtained using multivariate fMRI decoding techniques (local signals). In the second approach, we make use of neural responses in all of our 9 regions of interest to decode behavioral decisions by using a multivariate analysis that optimally combines information from the different brain regions (Fig. 3.1B). This enables us to obtain better decoding accuracy than by using each region separately, as well as to explore the relative contributions of each of these different areas to the final behavioral choice.
RESULTS

Local vs. global signals related to behavioral choice in regions of interest
To address the contribution of global vs. local signals in the encoding of behavioral choice, we separated the original fMRI data into global signals with a spatial scale bigger than 8mm, and local signals with a spatial scale smaller than 8mm (see Methods). Fig. 3.2A shows the statistical significance of each voxel when discriminating between switch vs. stay decisions in two subjects. Local signals (rightmost images) defined this way do not survive classical fMRI analysis (Fig. 3.2B), and can only be studied utilizing signal analysis techniques sensitive to spatially distributed signals. We evaluated the degree to which each individual region of interest could decode subjects’ subsequent choices, when using either global or local signals (Fig. 3.3A). Each subject underwent four separate fMRI sessions (70 trials each) during which they performed the decision making task. Four classifiers were trained and tested for each subject using four-fold cross validation, where each classifier is trained using three of the sessions (210 trials), and then tested on the session that is left out (70 trials). Decoding accuracy derived from global and local signals were comparable within each region of interest, suggesting that local and global signals strongly co-vary in each of the regions studied. There was a trend toward a greater contribution of local signals compared to global signals in overall decoding accuracy in ACC, although this did not reach statistical significance (at p<0.08)

Decoding accuracy of each individual region
When combining both local and global signals and evaluating decoding accuracy for each region alone, we find that each region can decode better than chance whether a subject is going to switch or not (Fig. 3.3B), with the highest accuracies being obtained by ACC (64%), anterior insula (62%), and DLPFC (60%). To address whether the difference in decoding accuracy across regions is merely a product of intrinsic differences in MR signal to noise in these areas, we examined the signal-to-noise ratio (SNR) in each region by analyzing responses elicited by the main effect of receiving an outcome compared to rest. All regions had comparable SNR to the main effect (Fig. S3.6), suggesting that accuracy
differences are unlikely to be accounted for by variations in intrinsic noise levels between regions.

**Combined accuracy across multiple regions**

Next, we aimed to determine whether a combination of specific brain regions would provide better decoding accuracy than just considering one region alone. For this we built regional classifiers using a multivariate approach that takes into account interactions between multiple regions of interest when decoding decisions (see Fig. 3.1B). This approach optimally combines both local and global signals from each region of interest. In order to determine which subset of regions to include in our classifier, we performed a hierarchical analysis whereby we started with the most accurate individual region, and then iteratively built multi-region classifiers by adding one region at a time. At each step in this iterative process, we added the region that increased the multi-region classifier’s accuracy the most (out of the remaining regions). Figure 3.3C shows the results of this process. We found that out of our nine regions of interest, a classifier with only 3 of these areas – ACC, mPFC, and ventral striatum – achieved an overall decoding accuracy of 67±2%, a significantly better decoding accuracy of subject’s choice than that provided by each region alone (for example, compared to ACC at p<0.01). Accuracy increase when adding regions is not only due to the signals related to behavioral choice in each region, but also depends on the degree of statistical independence of noise across regions (Fig. S3.4). Fig. 3.3D shows the average accuracy for each individual subject when using our region-based classifier. Receiver Operating Characteristic (ROC) curves representing the average classifier accuracy across a range of response thresholds are shown in Fig. S3.5 (see also Table S3.2).

Insula and DLPFC, which on their own have high decoding accuracy, were not selected in our hierarchical classifier, suggesting that signals from these regions are better accounted for by the other included regions. To account for the possibility that another combination of regions could substitute equally well for the regions included in the hierarchical classifier, we ran an additional analysis whereby we tested the classification accuracy of every possible combination of three regions (Fig. S3.7). Even in this case we still found that the
specific combination of regions identified from the hierarchical analysis were highest in decoding accuracy compared to all other possible combinations, supporting the conclusion that the specific regions we identified are sufficient for decoding decisions up to the overall decoding accuracy obtained in our study.

It should be noted that the approach we use here, whereby signals are combined across regions, proved significantly better at decoding behavioral decision making than alternative decoding techniques which do not employ this multi-region approach (Fig. S3.2). However, when we added more than 4 regions in the hierarchical analysis, the combined classifier’s accuracy gradually decreased again (Fig. 3.3C), perhaps due to over-fitting of the training data.

**Decisions per se or detection of rewarding vs. punishing outcomes?**

A key question is whether the decoding accuracy of our regional classifier is derived by detecting activity elicited by the decision process and its consequences, or merely reflects detection of the sensory and affective consequences of receiving a rewarding or punishing outcome on the preceding trial. To test this, we restricted input to the classifier to only those trials on which subjects received a punishing outcome. Even in this instance, the classifier was able to decode subjects’ decisions to switch or stay on the subsequent trial with $57\pm1\%$ accuracy, significantly better than chance (at $p<10^{-8}$, across subjects’ mean accuracy). This suggests that our classifier is using information relating to the behavioral decision itself and is not merely discriminating between rewarding or punishing outcomes on the immediately preceding trial. Additional analyses in support of this conclusion are detailed in the supplementary material at the end of this chapter.
DISCUSSION

The results of this study demonstrate that it is possible to decode with a high degree of accuracy, reward-related decisions or the consequence of those decisions (in terms of initiating a change in response choice) in human subjects on the basis of neural activity measured with fMRI before the specific physical action involved is either planned or executed. Theoretical accounts of goal-directed behavior differ in the degree to which decisions are suggested to be linked to the specific action needed to carry them out. According to one stimulus-driven view, decisions are computed abstractly in terms of the specific stimulus or goal the subject would like to attain\textsuperscript{115, 116}. In the case of the probabilistic reversal task used here, this comes down to a choice between which of two different fractal stimuli to select. An alternative approach is to propose that decisions are computed by choosing between the set of available physical actions that are required in order to attain a particular goal. Here, we measure neural responses on a preceding trial before subjects are presented with the explicit choice between two possible actions, and before subjects know which specific action they will need to select in order to implement that decision. The fact that these decisions can be decoded before subjects are aware of the specific action that needs to be performed to realize them (choose left button or right button), suggests that decision signals can be encoded in the brain at an abstract level, independently of the actual physical action with which they are ultimately linked\textsuperscript{115}. We should note however, that our decoding technique which is based on activity elicited at the time of receipt of the outcome on the preceding trial, is likely to be picking up both the decision itself and the consequence of the decision. In other words, once a decision to switch is computed, a change in stimulus-response mapping is going to be initiated, and the activity being detected in our analysis may also reflect this additional process.

Our findings have important implications not only for understanding what types of decisions are computed but also when these decisions are computed. In the context of the reversal task, it is possible for a decision to be computed at any point in time between receipt of the outcome on the previous trial, and implementation of the behavioral choice on the next. By using multivariate fMRI techniques we have been able to show that
subsequent decisions (or the consequences of those decisions) can be decoded on the basis of signals present on the preceding trials (after outcomes are received). This suggests that the decision to switch or maintain current response set may be initiated as soon as the information needed to compute the decision is available, rather than being implemented only when required on the subsequent trial.

In this study we also separated fMRI signals with a global spatial encoding from those with a local spatial encoding, and evaluated the information each contained for decoding behavioral choice within each of our regions of interest. Global signals, as we define them here, are relatively uniform spatially extended clusters of activation within a given area (with a spatial scale greater than 8mm). Typically, these spatially smoothed signals are those reported in conventional fMRI analyses, as they are especially likely to survive at the group level. However, recently it has been shown that information about task processes can be obtained from considering local spatially distributed variations in voxel activity\textsuperscript{105, 106}. In the present case, we defined spatially local signals as those with a spatial scale of less than 8mm. In this study we showed that within each region of interest, local signals do convey important information regarding behavioral choice over and above that conveyed by the global signals. However, we did not find strong evidence for a dissociation between regions in the degree to which they were involved in encoding local and global signals, except for a trend in anterior cingulate cortex toward a greater role for this region in encoding local as opposed to global signals. These results suggests that at least in the context of the present reversal learning task, the presence of global and local information relevant to behavioral decision making strongly covaries within areas. This is in contrast to results observed in the visual system, where in some instances local signals convey information pertaining to visual perception even when global signals do not. Local fMRI signals in visual cortex have been argued to relate to the columnar organization in this area of the brain. It should be noted however, that much less is known about the degree to which columnar organization exists outside of visual cortical areas, and hence, the underlying neural architecture that contributes to local fMRI signals in other areas of the brain such as the prefrontal cortex remains to be understood.
We also used a multivariate analysis technique, whereby the degree to which neural signals in multiple brain regions contribute to the decision process are evaluated simultaneously. This approach has allowed us to efficiently recruit signals from diverse brain regions in order to arrive at a better decoding accuracy for subjects’ behavioral choice than would follow from considering activity in any one region alone. As a consequence, our findings suggest that reward-related decision processes might be better understood as a product of computations performed across a distributed network of brain regions, rather than being the purview of any one single brain area.

Nevertheless, our results do suggest that some regions are more important than others. When we compared the decoding accuracy of classifiers incorporating information from all of our regions of interest to the accuracy of classifiers using information derived from different subsets of regions, we found that activity in a specific subset of our regions of interest accounted for the maximum accuracy of our classifier, namely the anterior cingulate cortex, medial PFC, and ventral striatum. Each of these regions have been identified previously as playing a role in decision making and behavioral choice on the basis of prior fMRI studies using traditional statistical analysis techniques. Out of these, one region in particular stood out as contributing the most: dorsal anterior cingulate cortex. This region has been previously implicated in diverse cognitive functions, including response conflict and error detection. However, a recent theoretical account has proposed a more general role for this region in guiding action selection for reward. While not incompatible with response-conflict or error-detection theories, our results are especially consistent with this latter hypothesis, suggesting that this region is playing a key role in implementing the behavioral decision itself.

Some of the regions featured in this study, such as dorsolateral prefrontal cortex, may contribute to task or cognitive-set switching more generally and are unlikely to be uniquely involved in reward-related decision making. However, it is notable that dorsolateral prefrontal cortex was ultimately not selected in our combined classifier. Instead, the regions that were selected have previously been specifically implicated in
reward-related learning and/or in implementing changes in behavior as a consequence of such learning, and not in cognitive set-shifting per se\textsuperscript{74, 86}.

The present study demonstrates that it is possible to decode abstract reward-related decisions in human subjects on a single trial basis, in essence by reading their decision before the action is executed. Our findings are consistent with the proposal that decision making is best thought of as an emergent property of interactions between a distributed network of brain areas rather than being computed in any one single brain region. Of all the regions we studied, we found that a subset of three regions seemed to contain information that was sufficient to decode behavioral decision making: ACC, medial PFC, and ventral striatum. Future studies are needed to determine whether these regions contain information specifically required for probabilistic reversal learning or whether other types of reward-related decisions can also be decoded on the basis of information contained in these areas.
MATERIALS AND METHODS

Subjects
Eight healthy, right-handed, normal subjects participated in this study (four female, mean age 27.6 ± 5.6 SD). The subjects were pre-assessed to exclude those with a prior history of neurological or psychiatric illness. All subjects gave informed consent and the study was approved by the Institute Review Board at Caltech. Subjects were paid according to their performance in the task. Before scanning, subjects were trained on three different versions of the probabilistic reversal task as in Chapter 1,\(^{109}\) (as in Chapter 1), which is also described in supplementary methods in this chapter.

Data acquisition and preprocessing
The functional imaging was conducted by using a Siemens 3.0 Tesla Trio MRI scanner to acquire gradient echo T2* weighted echo-planar images (EPI) with BOLD (blood oxygenation level dependent) contrast. To optimize functional sensitivity in OFC we used a tilted acquisition in an oblique orientation of 30° to the AC-PC line. Four sessions of 450 volumes each (4x15 minutes) were collected in an interleaved-ascending manner. The imaging parameters were: echo time, 30ms; field-of-view, 192mm; in-plane resolution and slice thickness, 3mm; TR, 2 seconds. Whole-brain, high-resolution T1-weighted structural scans (1x1x1mm) were acquired from each subject and co-registered with their mean EPI. Image analysis was performed using SPM2 (Wellcome Department of Imaging Neuroscience, Institute of Neurology, London, UK). To correct for subject motion, the images were realigned to the first volume, spatially normalized to a standard T2* template with a re-sampled voxel size of 3mm. Trials were 12 seconds long, and were time locked to the start of the fMRI EPI scan sequence. This was done to ensure that the scans from the previous trial used to decode the subject’s decision in the next trial would not be contaminated with BOLD activity arising from the choice itself on the subsequent trial. A running high-pass filter (the mean BOLD activity in the last 36 volumes, or 72 seconds, was subtracted from the activity of the current volume) was also applied to the data. This was used instead of the usual high-pass filtering\(^{96}\) so that BOLD activity in a volume would not be contaminated with activity from the choice itself in subsequent volumes. In the
scanner, visual input was provided with Restech (Resonance Technologies, Northridge, CA, USA) goggles, and subjects used a button box to choose a stimulus.

**Global and local spatial signals**

To dissociate global and local signals relevant to behavioral choice we used the following procedure: 1) The activity in each voxel was scaled such that the variance of the BOLD activity over all trials in a session was equalized across all voxels. 2) The fMRI data was spatially smoothed using a Gaussian kernel with a half width at half maximum (FWHM) of 8 mm, to capture global changes in signal. 3) fMRI data containing only locally distributed spatial signals were then extracted by subtracting the smoothed fMRI data (obtained in step 2) from the non-spatially smoothed fMRI data (obtained in step 1). This procedure adopts the assumption that BOLD activity is a function of the underlying neuronal activity that is identical across neighboring voxels, except for a scaling constant. Furthermore, step 1 estimates and eliminates the scaling differences across voxels, but errors in the estimation of this scaling could lead to an incomplete dissociation between local and global signals. The procedure also assumes that if local encodings exist, they will have the same spatial scaling characteristics across all brain regions.

**Region of interest (ROI) specification**

Nine regions of interest were specified based on previous literature implicating these regions in reward-related decision making, and delineated by anatomical landmarks (Fig. S3.1). Regions of interest were specified using a series of spheres centered at specified (x,y,z) MNI coordinates and with specified radii in millimeters (see Table S3.3 for complete specification).

**Discriminative analysis**

To optimally classify whether subjects will switch or stay in a given trial, the fMRI voxel activity \( \mathbf{x} \) (see Fig. 3.1A) is assigned to the action \( a_i \) for which the posterior probability \( p(a_i | \mathbf{x}) = p(\mathbf{x} | a_i) p(a_i) / p(\mathbf{x}) \) is maximal. Here \( p(\mathbf{x} | a_i) \) is the distribution of voxel activities given action \( a_i \). Assuming that the fMRI activity \( \mathbf{x} \) follows a multivariate normal
distribution with the same covariance matrix $\Sigma$ given either action, the posterior probability whether to choose the switch action is:

$$p(a_{\text{switch}} | x) = \frac{p(\text{switch}) e^{-\frac{1}{2}(x - \bar{x}_{\text{switch}})^T S^{-1} (x - \bar{x}_{\text{switch}})}}{p(\text{switch}) e^{-\frac{1}{2}(x - \bar{x}_{\text{switch}})^T S^{-1} (x - \bar{x}_{\text{switch}})} + p(\text{stay}) e^{-\frac{1}{2}(x - \bar{x}_{\text{stay}})^T S^{-1} (x - \bar{x}_{\text{stay}})}}$$

(3.1)

where $\bar{x}_{\text{switch}}$ and $\bar{x}_{\text{stay}}$ are the training sample means of the fMRI activity for both actions, and $S$ is the pooled covariance matrix estimated from the training sample. This can be simplified to $p(a_{\text{switch}} | x) = 1/(1 + e^{-y})$, where

$$y = w^T x - \theta, \quad w^T = (\bar{x}_{\text{switch}} - \bar{x}_{\text{stay}})^T S^{-1}$$

(3.2)

Here $\theta$ is a threshold variable that groups all constants. Given the brain voxel activity $x$ in a single trial, choosing the action with maximal posterior is equivalent to choosing the action for which $y > 0$.

The classifier was built in two steps (see Fig. 3.1B). In the first, nine regions of interest were specified (Fig. S3.1), and a unique signal from each region was obtained by adding up the activities of all voxels in that region, weighted by the voxels’ discriminability:

$$r = w^T x, \quad w_i = \frac{x_{\text{switch}} - x_{\text{stay}}}{\sigma_i^2}$$

(3.3)

where $\sigma_i^2$ is the pooled voxel variance of voxel i. This approach assumes that the noise is independent across voxels, a procedure used to avoid overfitting the classifier to the fMRI data. The second step utilizes the coalesced regional activities as input to a full Gaussian discriminative classifier (equation 3.2), where weights are assigned to each region.

Decoding accuracy is measured as the percentage of correctly decoded behavioral choices. That is, the mean between correctly decoded switch actions (number of correctly decoded switches divided by the total number of switch actions) and correctly decoded stay actions
(number of correctly decoded stays divided by the total number of stay actions). This measure takes into account the fact that the number of times a subject switches or stays can be different across sessions.
A Trial design

- ~.5s Subject makes choice
- 2s Fix.
- 3s Reward delivered
- 4s Fixation
- 12s Trial ends

B Region classifier construction

- within a region
  - Region signal
  - MPFC
  - mOFC
  - lOFC
  - Amygdala
  - vStriatum
  - dStriatum
  - DLPFC
  - Insula
  - ACC

- between regions
  - Decoding classifier
Figure 3.1. Reversal task setup and classifier construction. (A) Subjects chose one of two fractals, which on each trial were randomly placed to the left or right of the fixation cross. The chosen stimulus is illuminated until 2s after the trial onset. After a further 1s, a reward (winning 25 cents) or punishment (losing 25 cents) is delivered for 1s, with the total money earned displayed at the top. The screen is then cleared and a central fixation cross is then presented for 8s before the next trial begins. One stimulus is designated the correct stimulus in that choice of that stimulus leads to a monetary reward on 70% of occasions, and a monetary loss 30% of the time. The other stimulus is “incorrect,” in that choice of that stimulus leads to a reward 40% of the time and a punishment 60% of the time. After subjects choose the correct stimulus on 4 consecutive occasions, the contingencies reverse with a probability of 0.25 on each successive trial. Subjects have to infer the reversal took place and switch their choice, at which point the process is repeated. The last three scans in a trial are used by our classifier to decode whether subjects will switch their choice or not in the next trial. A canonical BOLD response elicited at the time of reward receipt is shown (in green) to illustrate the time-points in the trial at which the hemodynamic response is sampled for decoding purposes. A new trial was triggered every 12 seconds in order to ensure adequate separation of hemodynamic signals related to choices on consecutive trials. The average of three scans between the outcome of reward and the time of choice in the next trial was used for decoding subjects’ behavioral choice in the next trial. These three time-points will not only contain activity from the decision itself (activity taking place after the receipt of feedback, but before the next trial), but also activity from the reward/punishment received in the current trial, and activity consequent to the choice made in the current trial. (B) The multivariate region classifier used in this study is divided in two parts. The first extracts a representative signal from each region of interest (left box) by averaging the brain voxels within a region weighted by the voxels’ discriminability of the switch vs. stay conditions. To avoid overfitting the fMRI data, we did not take into consideration the correlations between voxels within a region of interest (equation 3.3, Methods). The second part of the classifier (right box) adds up the signal from each region, weighted by the region’s importance in classifying the subject’s decision (equation 3.2, Methods). Weights are calculated using a multivariate classifier that uses each region’s
decoding strength, and correlations between regions, to maximize the accuracy of the classifier in decoding whether subjects are going to switch or stay (see Methods – Discriminative analysis).
A  Global vs. Local activity

B  Random effects
Figure 3.2. **Global and local fMRI signals related to behavioral choice (A)** Here we show fMRI signals related to behavioral choice, i.e., whether subjects will switch or maintain (stay) their choices on a subsequent trial. Voxel t-scores for the discriminability between switch and stay trials is shown for 2 individual subjects with data in its original form (left-most panel), and then decomposed into a global spatial component (with spatial scale > 8mm, middle panel) and a local spatial component (spatial scale < 8mm, right panel). The ACC region of interest is outlined in white for reference. Red and yellow colors indicate increased responses on switch compared to stay trials, while blue colors indicate stronger responses on stay compared to switch trials. **(B)** Results from a group random effects analysis across subjects conducted separately for the original unsmoothed data, global data, and local data. While global signals survive at the random effects level (consistent with classical fMRI analyses), local spatial signals do not survive at the group random effects level. Random effect t-scores are shown with a threshold set at p<0.2 for visualization.
A  
Accuracy

B  
Individual Regions

C  
Regional Classifier Ordering

D  
Individual Subjects
Figure 3.3. Illustration of the decoding accuracy for subjects’ subsequent behavioral choices for each individual region and combination across regions (A) Plot of average accuracies across subjects shown separately for local and global spatial scales. Both spatial scales contain information that can be used to decode subjects’ subsequent behavioral choice in all of our regions of interest. Notably, decoding accuracies are comparable at the local and global scales within each region. (B) Plot of average across accuracy across subjects for each region individually combining both local and global signals. (C) Results of the hierarchical multi-region classifier analysis, averaged across subjects. An ordering of regions was performed by starting with a classifier which only contains the individual region with best overall accuracy (ACC, left column), and iteratively adding to this classifier the regions whose inclusion increases the accuracy of the classifier the most (or decreases the least). Thus, the second column shows the accuracy of a classifier containing ACC and ventral striatum, the third column the accuracy of a classifier containing ACC, ventral striatum, and mPFC, and so forth. The combination of the three regions which provide the best decoding accuracy are highlighted in grey. Addition of a fourth region (dorsal striatum) does not significantly increase decoding accuracy. All error bars indicate standard errors of the mean. (D) Decoding accuracy for the three-region classifier shown separately for each individual subject (Table S3.1).
MATERIALS AND METHODS

Pre-scan training
Before scanning, subjects were trained on three different versions of the task. The first is a simple version of the reversal task, in which one of the two fractals presented yields monetary rewards 100% of the time and the other monetary losses 100% of the time. These then reverse according to the same criteria as in the imaging experiment proper, as described in Fig 3.1A. This training phase is ended after the subject successfully completes 3 sequential reversals. The second training phase consists of the presentation of two stimuli that deliver probabilistic rewards and punishments as in the experiment, but where the contingencies do not reverse. The training ends after the subject consecutively chooses the ‘correct’ stimulus 10 times in a row. The final training phase consists of the same task parameters as in the actual imaging experiment. This phase ends after the subject successfully completes 2 sequential reversals. Different fractal stimuli were used in the training session than those used in the scanner. Subjects were informed that they would not receive remuneration for their performance during the training session.
SUPPLEMENTARY ANALYSES

Dissociating the contribution of reward vs. punishment signals on the immediately preceding trial from behavioral choice signals in the classification analysis

In order to determine whether our decision classifier was using information above and beyond that pertaining to receipt of rewards and punishers on the previous trial, we first explored how much information receiving a reward or a punishment on the previous trial provides about the subsequent behavioral choice. For this, we looked only at subjects’ reward and punishment histories (leaving aside their fMRI data), and trained a classifier to decode subjects’ subsequent choices on the basis of rewards and punishments received on an immediately preceding trial. This classifier was able to decode subjects choices with 63±1% accuracy, a decoding accuracy lower than that achieved by the classifier based on the fMRI data (of 67±2%, p<0.07 paired t-test across subjects). The decoding accuracy of the naïve classifier was due to the probabilistic nature of the task; in which subjects will sometimes switch and sometimes stay following receipt of a punishing outcome, but will always stay after receipt of a reward.

In addition to decoding behavioral decisions from brain activity, we also tested a classifier in order to determine whether on a given trial subjects received a reward or a punishment. This classifier (using the same three regions of interest as used for our decision classifier) was able to decode subjects’ receipt of a reward or punishment with 70±1% accuracy. This shows that these regions are sensitive not only to the actual behavioral decision, but also to whether subjects’ received a rewarding or a punishing outcome. Nevertheless, these findings also confirm that our decision classifier has to be using much more information than merely reading responses to the rewarding or punishing outcomes received on the previous trial, as the fact that rewards and punishments themselves can only be decoded up to an accuracy of 70% suggests that reward and punishment information on the immediately preceding trial probably constitutes a relatively minor contribution toward the overall accuracy of the decision classifier (because as shown above, even with 100% decoding accuracy, rewards and punishments on the previous trial can only decode 63% of subsequent decisions).
Figure S3.1. Brain regions of interest. Nine regions of interest were specified. These are: medial PFC, adjacent medial and lateral OFC, amygdala, ventral and dorsal striatum, left and right DLPFC, anterior insula, and ACC. fMRI signals in each region were used to decode subjects’ behavioral choices using a two-step multivariate classifier (see Fig. 3.1B). Table S3.3 specifies the coordinates of these regions of interest.
Accuracy

Three Regions Combined

Whole Brain Best Voxels

Accuracy

50% 55% 60% 65% 70%
Figure S3.2. Comparison of fMRI classifiers. Adding up all the brain voxels weighted by their discriminability (in decoding stay vs. switch decisions), which assumes voxels are independent of each other, yielded an accuracy of 61%. When taking the 20 individually best discriminating voxels and adding them independently, it yielded an accuracy of 64%. Our multivariate region-based classifier, which combines three regions of interest, yielded an average accuracy across subjects of 67% (blue column), significantly better than the other techniques (e.g., at p<0.04, paired t-test when compared to the 20 best voxel classifier). Error bars are fixed effects across subject sessions.
**Figure S3.3. Switch vs. stay for individual subjects.** Results of a simple linear contrast between signals in regions of interest derived from averaging the last 3 scans in a trial (after the receipt of outcome), according to whether subjects switch or stay on the subsequent trial, using smoothed data (global signals). The figures show t-scores within each region of interest that reflect the difference in signals between trials in which subjects subsequently switch their choice of stimulus to those trials where subjects stay (or maintain their previous choice of stimulus). Red and yellow colors indicate increased responses on switch compared to stay trials, while blue colors indicate stronger responses on stay compared to switch trials.
### Normalized Cross Correlation

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- **Color Scale**:
  - 0 (blue)
  - 0.2 (light blue)
  - 0.4 (green)
  - 0.6 (yellow)
  - 0.8 (orange)
  - 1 (red)

- **Normalized Cross Correlation Values**
Figure S3.4. Normalized cross-correlation of regression residuals across regions.
Regression residuals correspond to aspects of task processing that are not directly related to the subject’s behavioral decision; such as preparation of motor responses and stimulus processing. These processes can be shared between regions, thereby introducing correlations in time-courses between regions. The normalized cross-correlation matrix plot shown here illustrates the correlation between regions, after activity related to the decision has been removed. Each region is plotted on the vertical and horizontal axes, and correlations between any two regions can be found by finding the square corresponding to the intersection between the regions on these two axes. The color of each square depicts the intensity of the correlation, ranging from blue (uncorrelated) to red (fully correlated), see attached color scale. The diagonal elements express the correlation of the signal of a region with itself, and are by definition equal to one. The cross-correlation of residuals across regions shows a strong sharing of task processes between DLPFC, insula, and ACC (lower right triplet). In addition, ventral and dorsal striatum were found to be highly correlated in their activity. We also found weak correlations between residual activity in mPFC and medial OFC, and between DLPFC and OFC. In general, brain regions that are closer to each other in terms of physical distance, present higher inter-correlations than regions that are more distal, perhaps reflecting increased coupling between regions as a function of local connectivity.
Figure S3.5. Classification Receiver Operating Characteristic Curve. A measure used in signal detection to measure classifier discriminability is the area under a Receiver Operating Characteristic (ROC) curve\textsuperscript{120}, which represents the average classifier accuracy across all possible thresholds. Here we show typical ROC curves for the four sessions from one subject: blue for the session classifier on the training data, and red on the testing data. An area of 100 means the classifier will have on average an accuracy of 100\%, and an area of 50 means that on average the classifier will have a 50\% accuracy. Across subjects, ROC curves indicated an average accuracy of 71±2\% for decoding behavioral choices over all sessions (Table S3.2). The ROC curves of the classifiers for the training data and testing data are very similar, indicating that our procedure for training the discriminative classifier across regions of interest does not over-fit the fMRI data.
A - Switch-Stay Classifier

Avg. Normalized Weights

Avg. Norm. Weight Power

B - Region Main Effects

Avg. Region T-score

Avg. Region T-score Power

Available regions include mPFC, mOFC, LOFC, Amyg., vStriat., dStriat., DLPFC, Insula, ACC.
Figure S3.6. Behavioral choice responses and the signal-to-noise ratio in each region of interest. The fact that we find some regions to have higher decoding accuracy than others could reflect either a greater contribution of a given region toward computing a decision (and its consequences), or alternatively could be merely ascribed to global differences in the signal-to-noise ratio of the fMRI signal between regions. To account for this, we compared the sensitivity\(^{iv}\) of each region to the behavioral choice signals (top-right panel), with the sensitivity of each region as estimated from responses to the main effect at the time of outcome (bottom-right panel). As can be seen, SNR is comparable across regions of interest, and thus differences in behavioral choice sensitivity across regions are unlikely to be accounted for merely by an overall difference in SNR across these areas. The left panels show the average classifier normalized discriminant weights (t-scores) across subjects. Although some regions have classification power (sensitivity), they average out across individuals because they have different signs across regions.

\(^{iv}\) An unsigned mean sensitivity across subjects was obtained by 1) squaring the SNR per subject, 2) transforming to z-scores with an inverse F distribution 3) averaging z-scores across subjects 4) transforming back with an F distribution 4) taking the square root.
Accuracy

Combination of 3 regions of interest

Accuracy

Combination of 3 regions of interest
**Figure S3.7. Regional classifiers.** In Fig. 3.3D we created classifiers with an increasing number of regions to find the minimum set of regions that provided the maximum accuracy when decoding subjects’ decisions. However, the process might be misleading in that there might be a different combination of N regions that is better than a classifier with N regions built hierarchically. To test for this, we computed classifier accuracy for all combinations of three regions (out of 9). The accuracy of each combination of regions is shown ordered from left to right with the right most column corresponding to the best combination of regions (shown in black). This turns out to be ACC, ventral Striatum and mPFC, which enjoys a 1% advantage in decoding accuracy over the next best combination of regions. Thus, the results of this analysis accord with the hierarchical analysis in highlighting the three regions identified from that analysis as providing the most information for decoding behavioral choice out of all regions tested.
### Table S3.1. Multivariate classifier accuracy in decoding choice across subjects and subject sessions

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Table S3.2. Multivariate classifier ROC areas across subjects and subject sessions

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Table S3.3. Regions of Interest. Each region of interest is specified by a series of spheres centered at the specified (x, y, z) MNI coordinates and radius (mm).

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</table>
The prefrontal cortex receives substantial anatomical input from the amygdala, and these two structures have long been implicated in reward-related learning and decision making. Yet little is known about how these regions interact, especially in humans. We investigated the contribution of the amygdala to reward-related signals in prefrontal cortex by scanning two rare subjects with focal bilateral amygdala lesions using fMRI. The subjects performed a reversal learning task in which they first had to learn which of two choices was the more rewarding, and then flexibly switch their choice when contingencies changed. Compared to healthy controls, both amygdala lesion subjects showed a profound change in ventromedial prefrontal cortex activity associated with reward expectation and behavioral choice. These findings support a critical role for the human amygdala in establishing expected reward representations in prefrontal cortex, which in turn may be used to guide behavioral choice.

\[v\] Work done in collaboration with Ralph Adolphs, J. Mike Tyszka, and John P. O’Doherty.
INTRODUCTION

Research on the neural substrates of reward-related learning and decision-making has highlighted the important contributions of the ventromedial prefrontal cortex (vmPFC, encompassing the orbital and medial surfaces of the frontal lobes) and the amygdala. A large number of electrophysiology, lesion, and neuroimaging studies in humans and animals have examined the functions of these two structures. Lesion studies in rats and non-human primates suggest that both structures play an important role in learning associations between stimuli and subsequent reward or punishment, as well as in the adaptive control of behavior following changes in such reinforcement contingencies or in the value of the reinforcer. Single unit studies have found that neurons in both structures respond to stimulus cues predictive of future rewarding or punishing outcomes, or respond in anticipation of an impending outcome. Moreover, firing rates of these neurons track changes in reward contingencies over time, suggesting an important role for these regions in computing and rapidly updating reward expectations. Furthermore, lesion and neuroimaging studies in humans have also implicated amygdala and ventromedial prefrontal cortex in guiding behavioral choice under uncertainty, and have found evidence of neural activity related to expected reward and behavioral choice in both of these areas.

While much is now known about the involvement of amygdala and prefrontal cortex individually, these structures do not function in isolation, but as components of a network of brain structures important for reinforcement learning. The two structures are known to be bi-directionally connected anatomically, but very little is known about the functional significance of these connections. A small number of studies in animals have made use of the crossed-unilateral lesion technique to show that interactions between the two regions may be critical for certain reward-related functions, such as in the ability to modify behavior following a change in the value of an associated reinforcer. Electrophysiological studies in the vmPFC of rats have found that amygdala lesions substantially reduced the population of neurons in prefrontal cortex encoding expected outcomes, thus rendering these representations inflexible and stimulus-driven. The same
study also found a reduced number of neurons that were subsequently encoding the expected reward of choices made. These findings suggest that signals from the amygdala play an important role in facilitating neural representations of reward expectancy in vmPFC.

Much less is known about the functional significance of interactions between amygdala and vmPFC in the human brain. While some neuroimaging studies have begun to use connectivity analyses to model functional interactions between these regions, albeit not in the context of reward learning, the use of imaging techniques alone can provide only limited data about the causal effect of neural activity in one area on neural computations in another.

Here, we studied two rare human subjects with focal bilateral amygdala lesions due to Urbach-Wiethe disease (Fig. 4.1). The two subjects were scanned with fMRI while they participated in a task designed to probe reward-related learning and behavioral decision-making: monetary probabilistic reversal learning (Fig. 4.2). Previous studies have reported blood oxygenation-level dependent (BOLD) signal changes in both the amygdala and vmPFC that are related to processing rewarding and punishing outcomes in this task, and in encoding signals related to subsequent behavioral decisions. Moreover, activity in both of these regions tracks expected reward value during performance of this task, and these expectation signals are updated flexibly following changes in reinforcement contingencies.

We investigated the effects of amygdala lesions on reward representations in vmPFC by comparing the BOLD responses measured in the subjects with amygdala lesions to those measured in healthy control subjects. We looked for the effects of the amygdala lesion on BOLD signals correlated with behavioral choice (whether to maintain current choices or switch choices in the task), computation of expected reward value (how much money they expected to earn or lose following their choice), and value of the outcomes (the actual monetary gain or loss at the end of each trial). We hypothesized that the amygdala
contributes to computations of expected reward value in vmPFC, which in turn should affect signals of behavioral choice.
RESULTS

Subjects
Both amygdala subjects had focal bilateral lesions in the amygdala due to Urbach-Wiethe disease (Fig. 4.1). One of the subjects, S.M., has been extensively published before: she is a 41-year-old woman with a high-school education, IQ in the normal range, and normal basic visuoperception, language, and memory; her lesions encompass the entire amygdalae, as well as subjacent white matter and very anterior entorhinal cortex. The second subject, A.P., is a 21-year-old woman in college with likewise normal IQ, perception, language, and memory; lesions are entirely confined to the amygdala, occupying roughly 50% of each amygdala’s volume. Both subjects are fully right-handed, live independently, and show no evidence of psychopathology on tests of personality assessment. Both subjects also perform normally on standard neuropsychological tests of response switching, such as the Wisconsin Card Sorting Task and the Trailmaking task. Subjects with amygdala lesions were compared to 16 healthy controls of similar age as A.P. (Controls), as well as to 4 healthy women similar to S.M. in age (SM-comparisons, see Methods for further details).

Behavioral performance on probabilistic reversal learning

Subject AP
AP was significantly impaired relative to controls in the average number of trials to reach the criterion for contingency reversal, and hence impaired in the number of contingency reversals obtained over the course of the task. AP obtained only 3 reversals, whereas controls obtained on average 8 reversals, a performance 3.5 S.D.s below the control mean (p<0.002). A further analysis of AP’s performance revealed that, although she was not significantly more likely to switch choice when compared to controls (Fig. 4.3), her choice of when to switch was inconsistent with the reward contingency: in particular, she switched choice after obtaining a positive reward much more often than did the controls. For this reason, although she switched the same amount as controls did overall, she achieved fewer contingency reversals in the task.
**Subject SM**
SM was also impaired relative to controls in the number of contingency reversals, obtaining 4 reversals. However, the SM-comparison subjects who were closer in age to SM were also variably impaired relative to the control group, and when SM’s data were compared to this group of subjects of more similar age, her behavioral variables did not stand out as abnormal (Fig. 4.3). It should be noted that for the purpose of our fMRI study, the fact that SM performs similarly to SM-comparison subjects is in fact advantageous, as any observed differences in BOLD signal would then be unlikely to be confounded by differences in behavior\textsuperscript{135}.

Both amygdala lesion subjects were also impaired in the number of trials taken to reach acquisition during the initial training phase outside the scanner (even before reversals took place). However, as in the behavioral performance during scanning itself, the SM-comparison subjects were also impaired relative to the younger controls (see Fig. S4.1).

**fMRI Results**
Here we report whole brain analyses of BOLD signal correlating with parameters from our models that are based on the behavior of the control subjects, but restrict our analysis of the difference in BOLD signal between amygdala lesion subjects and controls to those regions that originally elicited a signal in controls—notably, specific regions of prefrontal cortex (see Methods). fMRI data for this same set of control subjects is more extensively analyzed in Hampton et al.\textsuperscript{109}.

**Behavioral choice signals**
In order to determine the effects of amygdala lesions on BOLD signals in orbital and medial prefrontal cortex related to behavioral choice, we first conducted a simple canonical trial-based analysis of the fMRI data whereby we examined BOLD responses following receipt of the outcome on a given trial (as in O’Doherty et al.\textsuperscript{74}). Trials were separated according to whether on the subsequent trial following the outcome subjects changed their choice of stimulus (“switch” trials) or continued choosing the same stimulus (“stay” trials).
Fig. 4.4A shows areas with significant responses in “switch” compared to “stay” trials in control subjects. This contrast revealed significantly greater activity in “switch” compared to “stay” in anterior frontal insula, extending into posterior lateral orbitofrontal cortex and anterior cingulate cortex (ACC). The reverse contrast revealed significant effects in mPFC (Fig. 4B). These results are consistent with previous studies of reversal learning in healthy control subjects.

Differences in behavioral choice signals in amygdala subjects compared to controls

We examined regions in which the above contrast would differ between our two subjects with amygdala lesions and controls by restricting the analysis to those voxels that showed a significant effect in the controls in the first place (for switch-stay; Fig. 4.4A). We found significantly greater responses in switch compared to stay trials in control subjects than in the two amygdala subjects in a region of posterior lateral orbitofrontal cortex/anterior insula, bilaterally (Fig. 4.4C, threshold at p<0.01). These differences were significant in each amygdala subject individually compared to controls (at p<0.001 for SM and at p<10^-8 for AP). A plot of the contrast estimates for switch-stay are shown in Fig. 4.4D. It is notable that responses in both amygdala subjects are markedly different from controls, and even from the SM-comparison subjects (who were similar in their behavioral performance to SM). A comparison of the reverse contrast (stay-switch) between amygdala subjects and controls did not reveal any significantly decreased responses in the amygdala subjects.

These results indicate that bilateral damage to the amygdala results in altered responses in anterior insula/posterior orbitofrontal cortex and anterior cingulate cortex related to behavioral choice, suggesting that in healthy individuals the amygdala makes an important contribution to the computation of behavioral control signals in those regions.

Expected reward signals

We next examined BOLD responses to expected reward. For this, we applied a computational model which calculates expected reward signals related to subjects’ choice in a trial by taking into account the history of rewards and punishments obtained and the history of choices made (see Methods). In our control subjects, we found significant
correlations with this signal in orbital and medial prefrontal cortex (Fig. 4.5A), time locked to the time of choice. Activity in these areas increases in a linear fashion as a function of increasing expected reward value\(^{109}\) suggesting that these areas are involved in encoding the expected reward of the currently chosen stimulus.

**Differences in expected reward signals between amygdala subjects and controls**

In a direct comparison between areas correlating with expected reward signals in the amygdala subjects and controls, we found significant differences in medial prefrontal cortex at \(p<0.001\) (Fig. 4.5B). These results were significant in each subject individually when compared to controls at \(p<0.001\) for AP and \(p<0.0001\) for SM. A consistent difference between AP and controls, and between SM and SM-comparison subjects can be seen when plotting the regression coefficients of all subjects in medial prefrontal cortex (Fig. 4.5C), confirming that the amygdala subjects process the expected reward value of each choice abnormally. These results were obtained by fitting a model to the behavior of the group of 16 controls, and then using the model parameters as the regressor against the fMRI data from the amygdala subjects, as well as the fMRI data from the controls. However, in order to account for the possibility that a difference in model parameters between the controls and amygdala subjects could account for the above results, we also performed the same analysis using parameter fits derived individually from each of the amygdala subjects. This analysis yielded the same results: a significant difference in expected reward signals in medial prefrontal cortex in amygdala subjects compared to controls (Fig. S4.2A).

To further characterize how amygdala lesion subjects process expected reward representations in medial prefrontal cortex, we plotted the signal in medial PFC measured with fMRI against the expected reward signals obtained from the model of the subjects’ task performance. We sorted trials into one of 5 bins to capture different ranges in the expected reward values and fitted each bin separately to the fMRI data. For controls, this analysis shows a linear increasing relationship between the magnitude of the evoked fMRI signal in this region and expected reward value. By contrast, responses in mPFC in the
amygdala subjects did not display a clear linear increasing relationship with expected reward (Fig. 4.5D).

**Responses to rewarding and punishing outcomes**

We also looked for responses relating to the receipt of rewarding or punishing feedback at the time of outcome. In our control subjects, when comparing responses to receipt of rewarding compared to punishing outcomes, we found significant activity relating to receipt of reward in medial prefrontal and medial orbitofrontal cortex (Fig. 4.6A), consistent with previous reports. On the other hand, when testing for areas responding to punishing compared to rewarding outcomes we found significant effects in the anterior ventrolateral prefrontal cortex extending into lateral OFC, also consistent with previous results.

**Differences in responses to rewarding and punishing outcomes in amygdala subjects compared to controls**

We then compared the above contrast in amygdala subjects to that in the control subjects, again restricting ourselves to those regions that showed significant effects (at p<0.01) of rewarding or punishing feedback in the control subjects in the first place. We found no significant differences in BOLD responses to rewarding or punishing feedback in amygdala subjects compared to controls at p<0.001 uncorrected, with only a single voxel surviving in medial PFC in the reward contrast at p<0.01 (Fig. 4.6B). These results suggest that processing of rewarding and punishing feedback in OFC and medial PFC remains intact after amygdala lesions. Thus, amygdala lesions appear to impair selectively the generation of behavioral choice and expected reward signals in prefrontal cortex, but leave the generation of reward outcome signals relatively unaffected.

**Controlling for behavioral differences between amygdala subjects and controls**

In order to further control for the possibility that differences in behavior between the amygdala subjects and controls could contribute to the results observed, we performed a follow-up analysis in which we selected only those trials on which every subject had made a correct choice according to the underlying task contingency. That is, we chose those trials
on which subjects correctly maintained their choice of stimulus (if their current choice of stimulus was correct), and those trials on which subjects correctly switched their choice of stimulus after the contingencies had reversed. All other trials were modeled separately as error trials of no interest. We then conducted the same analyses reported above for each contrast of interest. All of the above results held up (see Figs. S4.2B and S4.3), indicating that the abnormal signal in prefrontal cortex that we report following amygdala damage cannot be due simply to differences in the distribution of errors made between controls and amygdala subjects.
DISCUSSION

Amygdala and ventromedial prefrontal cortex are known to play an important role in reward-related learning and decision making, yet little is known about how these structures interact to support such functions in the human brain. In the present study, we provide novel evidence that neural representations in orbital, medial, and lateral prefrontal cortex related to the computation of expected reward and the computation of behavioral choice depend on input from the amygdala. Moreover, our results indicate that reward outcome representations in vmPFC are not as dependant on amygdala input.

Consistent with previous reports\textsuperscript{74, 75}, we found robust signals related to behavioral choice in posterolateral OFC, agranular insula, and anterior cingulate cortex in healthy individuals. By contrast, these signals were significantly reduced in both subjects with amygdala lesions. Moreover, this effect is unlikely to be driven by differences in behavioral performance between the amygdala subjects and their controls, as these results held up even when behavioral differences between the patients and controls were taken into account in the fMRI analysis (by restricting analysis to only those trials in which both amygdala subjects and controls made the correct choices). Furthermore, subject SM did not show overall behavioral impairments relative to her age-matched comparison subjects, yet still showed significant differences in activity in the target regions of prefrontal cortex compared to controls. Thus, differences in neural signals in this area are unlikely to be merely a consequence of the degree of behavioral impairment on the task, but are likely to be a direct consequence of the amygdala lesion. These results support the hypothesis that the production of signals related to behavioral choice in OFC and anterior cingulate cortex relies directly on input from the amygdala.

This conclusion leaves open the question of what precisely the amygdala contributes to behavioral choice signals in prefrontal cortex. Computational models of decision-making such as those grounded in reinforcement learning approaches, conceive of behavioral decision-making as being driven by an underlying computation of expected rewards or utilities for different available actions or stimuli. Decisions are then weighted according to the relative value of the different actions, so that over the course of learning choices
associated with higher value become favored (with the caveat that actions believed to be sub-optimal nonetheless may sometimes be selected for the purposes of exploration\textsuperscript{17}). The decision process is likely therefore to involve an explicit comparison between expected reward values available for different actions. In the case of reversal learning, there are only two possible actions: either maintaining current behavioral choice when the chosen stimulus is believed to be correct, or switching stimulus choice once a change in contingencies has been detected. Here we used a computational model of decision-making, which is essentially a modified reinforcement learning algorithm that also takes into account the reversal structure of the task. This model computes expected reward signals based on the history of prior outcomes. Previously we have shown that BOLD signals in ventromedial prefrontal cortex reflect computations of expected reward according to this model\textsuperscript{109}. We hypothesize that these expected reward signals are then used as input to the decision-making process in order to determine whether to maintain current stimulus choice or switch stimulus choice in the task.

In the present study we found that expected reward signals in medial PFC were markedly abnormal in the amygdala lesion subjects. Whereas control subjects showed a linear increase in activity in this region as a function of increased expected reward value, no such relationship was found in the subjects with amygdala lesions. The absence of normal expected reward signals in the medial PFC of subjects with amygdala lesions implies that these signals can no longer be used appropriately to generate behavioral decisions. The lack of these expected reward signals could therefore also account for the difference in observed behavioral choice signals. Thus, we suggest that the primary contribution of amygdala-vmPFC interactions is in computing expected reward values which, once established, are then used to generate behavioral decisions.

While we found significant effects of amygdala lesions on prefrontal signals of expected reward and behavioral choice, we found no such effects on signals of receipt of the outcome. In control subjects, receipt of monetary reward elicited robust signal in medial prefrontal cortex extending down to the medial orbital surface, consistent with many previous findings\textsuperscript{53, 137, 139}. However, when comparing BOLD signal in controls to that in
amygdala lesion subjects, we found no significant differences, except for a single voxel at p<0.01, suggesting that differential processing of reward feedback in this area is unaffected by the lesion. Similarly, BOLD signal to punishing feedback was found in lateral areas of prefrontal cortex (on the lateral surface and extending down to lateral OFC) in controls, again consistent with prior observations. However, once again there were no differences between activity in amygdala subjects and controls in these responses. Thus, our findings indicate that amygdala lesions selectively impair some but not all aspects of reward-related processing in vmPFC, ruling out a non-specific effect of amygdala lesions on vmPFC function or on the BOLD signal in general.

To conclude, the results of the present study highlight an important contribution of amygdala-vmPFC interactions toward the computation of expected reward value in humans, and support a model of decision making whereby these expected reward signals, once computed, are integrated in vmPFC and then subsequently used to guide behavioral decision making. More generally, these results highlight the utility of combining studies of human subjects who have discrete lesions with neuroimaging in order to address computationally-driven hypotheses about the functional significance of neural interactions between brain areas. While the present study has addressed the role of amygdala lesions on vmPFC function, a fruitful avenue for future research will be to investigate the converse effects of vmPFC lesions on amygdala function, and to explore interactions with additional structures involved in reward processing, such as the ventral striatum.
MATERIALS AND METHODS

Subjects
Two subjects with bilateral amygdala lesions (AP: age 20, Full-Scale IQ 98, VIQ 92, PIQ 106; and SM: age 42, Full-Scale IQ 88, VIQ 86, PIQ 95) participated in this study. Sixteen healthy, normal subjects also participated as controls (8 female, 14 right handed), as well as four subjects similar to SM in age and IQ (all female, mean age 53±7, mean IQ 99±4). Control subjects excluded those with a prior history of neurological or psychiatric illness. All subjects gave informed consent and the study was approved by the Institutional Review Board at Caltech. Before entering the scanner, subjects were informed that they would receive what they earned (or lost) in the task, added to an initial amount of $25 dollars. It was not possible for subjects to produce a net monetary loss in the study.

Pre-scan training
Before scanning the subjects were trained on three different versions of the task. The first is a simple version of the reversal task, in which one of the two fractals presented yields monetary rewards 100% of the time and the other monetary losses 100% of the time. These then reverse according to the same criteria as in the imaging experiment proper, where a reversal is triggered with probability 0.25 after 4 consecutive choices of the correct stimulus. This training phase is ended after subjects successfully complete 3 sequential reversals. The second training phase consists of the presentation of two stimuli that deliver probabilistic rewards and punishments as in the experiment (see Fig. 4.2), but where the contingencies do not reverse. The training ends after the subject consecutively chooses the “correct” stimulus 10 times in a row. The final training phase consists of the same task parameters as in the actual imaging experiment (stochastic rewards and punishments as described in the main text, and stochastic reversals). This phase ends after the subject successfully completes 2 sequential reversals. Different fractal stimuli were used in the training session than those used in the scanner. Subjects were informed that they would not receive remuneration for their performance during the training session. Subject performance during training can be seen in Fig. S4.1.
Data acquisition and pre-processing

Blood oxygenation level dependent (BOLD) functional MRI was conducted using a Siemens 3.0 Tesla Trio MRI scanner to acquire gradient echo T2* weighted echo-planar images (EPI). An eight-channel phased array head coil was used for radiofrequency reception. Visual stimuli were presented using Restech (Resonance Technologies, Northridge, CA, USA) goggles, and subject responses were recorded with a button box. Oblique axial-coronal slices were acquired at 30° to the AC-PC line for a neutral head position to minimize signal loss and geometric distortion in the orbitofrontal cortex (OFC). A total of 580 volumes (19 minutes) were collected during the experiment in an interleaved-ascending slice order. The imaging parameters were: echo time (TE), 30ms; field-of-view (FOV), 192mm; in-plane resolution and slice thickness, 3mm; repetition time (TR), 2 seconds. Whole-brain, high-resolution T1-weighted structural scans (1x1x1mm) were also acquired from the control subjects, co-registered with their mean EPI and averaged to permit anatomical localization of the functional activations at the group level. Image analysis was performed using SPM2 (Wellcome Department of Imaging Neuroscience, Institute of Neurology, London, UK). Temporal normalization was applied to the scans, each slice being centered to the middle of the scan (TR/2). To correct for subject motion, all EPI volumes were realigned to the first volume, spatially normalized to a standard T2* template with a resampled voxel size of 3mm, and spatially smoothed using a Gaussian kernel with a full width at half maximum (FWHM) of 8mm. Intensity normalization and high-pass temporal filtering (using a filter width of 128 secs) were also applied to the data. The same process was applied to the amygdala lesion subjects, and no qualitative spatial distortion effects due to the normalization process could be seen near the lesioned area in either the functional EPI or structural T1-weighted scans.

Data Analysis

Behavioral Data

To compare the behavior of the amygdala lesion subjects in the probabilistic reversal task to control subjects, we analyzed two measures: First, how often do subjects switch choice (depending on whether they have just received a reward or a punishment); and second, how many task contingency reversals does their behavior elicit, given that subjects have to reach
the criterion of making the correct choice four times in a row before task contingency probabilistically reverses. For statistical comparisons across subjects, these measurements are assumed to have a binomial distribution for each individual subject, and are modeled as a beta distribution across subjects (the beta distribution being a conjugate prior distribution of the binomial distribution\textsuperscript{142}). Behavioral statistical significance t-scores (Fig. 4.3) of the difference between each amygdala lesion subject and control subjects were appropriately corrected to take into account that measurements have a beta distribution instead of a Gaussian distribution.

\textit{Computational model-based analysis: generating expected reward signals}

In order to generate signals related to subjects’ expected reward value on each trial we used an approximation to the Hidden Markov Model formulation used previously\textsuperscript{109}, whereby in order to choose optimally, it is necessary to compute expected reward signals not only by taking into account the history of rewards and punishments received on a given choice, but also the structure of the task: namely, that when one choice is correct, the other is not (this derivation is further detailed in Appendix B). Rewards and punishments received on each trial were used to update both the selected and unselected choices. Thus, after making choice \( A \) and receiving a reward, the update of the value of both choices becomes:

\[
V_{A}^{t+1} = V_{A}^{t} + \eta(R_{t} - V_{A}^{t}) \\
V_{B}^{t+1} = V_{B}^{t} + \eta(-R_{t} - V_{B}^{t})
\]

(4.1)

where \( R_{t} - V_{A}^{t} \) is the prediction error between the reward \( R_{t} \) subjects obtained at time \( t \), and the expected reward \( V_{A}^{t} \) of their choice. This model is therefore a variant of standard reinforcement learning, except for the additional updating of the action not taken (action B), similar to fictive updating in RL\textsuperscript{143, 144}. This model states that subjects assume that the reward they would have received for the choice not taken is exactly opposite to the reward they receive for their current choice. Although reward outcomes are probabilistic, this update correctly captures the anti-correlation between choice values in this task.
To choose which action to make (A or B), the model compares their expected rewards to select which will give the most reward in the future. The probability of choosing action $A$ is:

$$P(A) = \sigma(\beta [(V_B - V_A) - \alpha])$$

(4.2)

where $\sigma(z) = 1/(1 + \exp(z))$ is the Luce choice rule$^{18}$ or logistic sigmoid, $\alpha$ indicates the indecision point (when it’s equiprobable to make either choice) and $\beta$ reflects the degree of stochasticity in making the choice (i.e., the exploration/exploitation parameter).

In order to estimate the free parameters in the model, we fit the model predictions to subjects’ actual behavior data, and selected those parameters which minimized the error in the fit of the model to the behavioral data (using logistic log likelihood errors). We used the multivariate constrained minimization function (fmincon) of the Optimization Toolbox 2.2 in Matlab 6.5 (www.mathworks.com) for this fitting procedure.

**FMRI data analysis**

**Behavioral choice**

For the analysis of behavioral choice signals, we conducted an analysis similar to that reported in O’Doherty et al.$^{74}$. For this, we categorized trials according to subjects’ reward outcomes and subsequent behavioral choices. We modeled event-related responses at the time of receipt of the outcome, and differentiated between trials in which subjects subsequently switched their choice of stimulus (switch trials), and trials in which subjects maintained their current choice of stimulus (stay trials). These two type of trials were further differentiated by whether subjects received a punishment or a reward as a consequence of their choice in the current trial. Separate regressors were entered for reward-stay, reward-switch, punish-stay, and punish-switch trials, by constructing sets of delta (stick) functions at the time of the outcome for each trial type. A common regressor across all trial types was also modeled at the time of choice. These regressors were then convolved with a canonical hemodynamic response function. In addition, the 6 scan-to-scan motion parameters produced during realignment were included to account for residual
motion effects. These regressors were fitted to each subject’s fMRI data individually, and the regression parameters were then taken to the random effects level, to generate group random effects statistics. The regression parameters for both amygdala subjects were modeled separately at the random effects level from the regression parameters for the control subjects. A linear contrast was then computed between the amygdala subjects and controls to identify areas showing significant differences between amygdala subjects and controls. For the results reported in the present study, we tested for areas showing significantly decreased responses in the amygdala subjects compared to the controls at $p<0.001$ uncorrected in our regions of interest, restricted to those areas showing significant effects for the switch-stay contrast in the control subjects (at $p<0.01$ or lower). The results were also masked to show only those voxels that are significantly different to controls in each amygdala lesion subject individually (at $p<0.05$ or lower), in order to select only those areas that are significantly different in both subjects compared to controls.

**Expected reward signals**

We then conducted an additional analysis to detect brain regions correlating with expected reward. For this, regressors were constructed using the trial-by-trial expected reward signals as predicted by the computational model described above, given the trial history of each individual subject. These were then entered as parametric regressors set at the time of choice. We also modeled the outcome received on each trial (whether a reward or a punishment was obtained). As before, these regressors were convolved with a hemodynamic response function, and motion regressors were included as effects of no interest.

These regression fits were then taken to the random effects level separately for the contrasts of expected reward signals at the time of choice and rewards vs. punishments at the time of outcome, and a comparison was computed between both amygdala subjects and controls for each contrast separately. Statistical significance was reported at $p<0.001$ uncorrected in our regions of interest. As before, we restricted our analysis to those voxels showing significant effects in the relevant contrast in the controls (at $p<0.01$ or below), and show
only those voxels that surviving a comparison of each individual amygdala subject to controls significant at p<0.05 or lower.
Figure 4.1. Axial T1-weighted structural MR images from the two amygdala lesion subjects. Selective bilateral calcification of the amygdala (arrows) due to Urbach-Wiethe disease\textsuperscript{134} is evident as loss of signal on these T1-weighted structural MR scans of the brains of S.M. (left) and A.P. (middle). An image from a typical healthy control subject with intact amygdalae is also shown for comparison (right). Multiple axial slices for both amygdala lesion subjects are shown in Fig. S4.4.
Trial Onset

~500ms
Subject makes choice

5000ms
Reward delivered

7000ms Trial ends
Figure 4.2. Probabilistic reversal task. Subjects chose one of two fractals, which on each trial were randomly located to the left or right of a fixation cross. Once a stimulus was selected by the subject, it increased in brightness and remained on the screen for a total of 2 seconds. After a further 3s blank screen, a reward (winning 25 cents, depicted by a quarter dollar coin) or punishment (losing 25 cents, depicted by a quarter dollar coin covered by a red cross) was shown, with the total money earned displayed at the top ($2.50 in this figure). One stimulus was designated as the correct stimulus and resulted in a monetary reward on 70% of occasions, and a monetary loss 30% of the time, with an overall accumulation of monetary gain in the task. The other, “incorrect” stimulus resulted in a reward 40% of the time and a punishment 60% of the time, with a cumulative monetary loss. After subjects chose the correct stimulus on 4 consecutive occasions, the contingencies reversed with a probability of 0.25 on each successive trial. Subjects had to infer that the reversal took place and switch their choice, at which point the process was repeated.
Figure 4.3. Behavioral performance. During the reversal task performed in the scanner, both AP and SM took more trials to reach reversal criteria than controls (left panel – AP $p<.002$, SM $p<.02$). This was not directly caused by the number of choice switches the amygdala subjects executed (right panel), for AP did not significantly switch more often than controls did. However, the “quality” of these decisions was arguably impaired, in that the amygdala subjects would switch choice more often than controls in situations when it was not advantageous to do so, such as switching choice after receiving a rewarding outcome. However, controls of similar age to SM (right column in both panels; bar shows means, yellow symbols show individual data) were also impaired on these measures, compared to the younger controls.
A Switch-stay controls

B Stay-switch controls

C Controls > amygdala (sw-st)

D Switch-Stay Contrast

- Controls
- SM Com.
- SM
- AP
**Figure 4.4. Behavioral choice signals.** Contrast (aligned to the time of outcome) between trials for which subjects subsequently switch their choice of stimulus (“switch”), compared to trials for which subjects subsequently continue choosing the current stimulus (“stay”).

(A) Regions showing increased BOLD signal on “switch” compared to “stay” trials in control subjects. Significant effects were observed in anterior frontal-insula/posterior lateral orbitofrontal cortex bilaterally (-30, 21, -9mm, z=3.91 and 33, 21, -12mm, z=3.64), anterior cingulate cortex (-9, 21, 33mm, z=3.62), and extending into pre-motor cortex (0, 18, 51mm, z=3.73).

(B) Regions showing increased BOLD signal on stay compared to switch trials. Significant effects were observed in mPFC (-6, 45, 21mm, z=3.79).

(C) Both amygdala subjects had significantly less switch vs. stay activity than controls in posterior lateral orbitofrontal cortex/anterior insula bilaterally (-30, 21, -18mm, z=4.2 and 36, 21, -18mm, z=4.32) and ACC (-9, 33, 42mm, z=5.29).

(D) Switch-stay contrasts in both these areas show that not only are both amygdala subjects significantly different from the 16 younger controls as well as the SM-comparison subjects, but also that the control group shows a very similar signal to the SM-comparisons. This is in stark contrast with the behavioral results, where SM-comparisons were also impaired when compared to the younger controls. Thus, behavioral differences are not driving the differences in fMRI signal.
A  Expected reward (controls)

B  Controls > amygdala (exp. rew.)

C  Regression Coefficient

D  Expected reward in mPFC
**Figure 4.5. Expected reward signals in the brain.** (A) For control subjects, BOLD signal correlating with the magnitude of expected reward of a choice was found in ventromedial prefrontal cortex (6, 57, -6mm, z=5.13) and the amygdala bilaterally (-27, -6, -21mm, z=3.89) extending into hippocampus. (B) We found a significantly weaker correlation with expected reward signal in mPFC (6, 57, -3mm, z=4.12) in the two amygdala subjects compared to controls (at p<0.001). (C) Regression coefficients in mPFC (6, 57, -3mm) indicate that both amygdala subjects differ markedly from controls in their representation of expected rewards, and SM differs with respect to SM-comparison subjects as well. (D) To analyze the relationship between expected rewards and BOLD signal in medial PFC, we subdivided trials into five bins depending on the expected reward value on that trial. The regression coefficients for each bin are plotted for the control subjects and for the SM-comparison subjects (left panel), showing the linear relationship between expected rewards and brain BOLD activity. However, in contrast to the controls, the relationship between expected rewards and BOLD activity for both amygdala lesion subjects is nearly flat, indicating that both subjects are not computing expected rewards in mPFC in the same way as controls. Regression coefficients were extracted at the local maximum of the expected reward contrast for each subject, within a 10 mm radius of the group peak (as shown in panel B).
A  Controls

Rew - Pun  Pun - Rew

Y=+52  Y=+52

B  Controls > amygdala

Rew - Pun  Pun - Rew

Y=+57

\[ p < .01 \]

\[ p < .001 \]
Figure 4.6. Responses to receipt of rewarding and punishing outcomes. (A) In a direct comparison of BOLD responses to rewarding and punishing outcomes in control subjects, we found increased activity in medial OFC to receipt of rewarding compared to punishing outcomes (-3, 57, -9mm, z=4.23), and increased activity in anterior ventrolateral prefrontal cortex extending into far lateral OFC to the receipt of punishing outcomes (27, 52, 6mm, z=3.45). (B) However, in a direct comparison of responses to rewarding outcomes between the amygdala lesion subjects and controls, we found no significant differences (except one voxel at p<0.01 in mPFC). Similarly, no differences were found in BOLD signal responses to punishing outcomes between amygdala subjects and controls. This suggests that outcome representations in orbital, medial, and lateral prefrontal cortices are unaffected by the amygdala lesions.
Acquisition performance

Trials to reach criteria

Number of switches

T-score

AP  SM  SM Com.

AP  SM  SM Com.
Figure S4.1. Subject performance during task acquisition. During training, subjects were exposed to a version of the task where one choice was always on average rewarding while the other was always on average punishing (see Methods). Both amygdala lesion subjects took a larger number of trials to reach the criteria of choosing the correct stimulus (by making the correct choice ten times in a row, or reaching the maximum number of trials allocated to this training phase). This is shown on the left panel, which shows a comparison of the number of trials taken to finish training for amygdala subjects and SM-comparison subjects (individual subjects marked with yellow rhomboids), when compared to controls. Furthermore, amygdala subjects and SM-comparison subjects switched their choice of stimulus more often on average than controls did (right panel).
Controls > amygdala (expected reward)

A  Individual model parameters

B  Excluding all error trials
Figure S4.2. Controlling for the effects of behavioral differences between amygdala subjects and controls on signals pertaining to expected reward value. To control for the effects of differences in behavioral performance between the amygdala subjects and control subjects on the fMRI results for the comparison of expected reward signals (Fig. 4.5B), we performed two additional tests: (A) We first compared expected reward signals between the amygdala subjects and controls, but this time with model parameters derived from the best log likelihood fits of the computational model to the behavioral data for each amygdala subject individually. This controls for the possibility that the model accounts equally well for the behavioral data in the amygdala lesion subjects as the controls, but that the amygdala subjects and controls only differ in the model-parameters. Contrary to this possibility, this analysis still revealed significant differences between amygdala subjects and controls in expected reward signals (again at p<0.001), again consistent with the results reported in the this chapter. (B) We then compared expected reward signals between the two amygdala subjects and controls using only those trials for which subjects made correct choices given the underlying contingencies. Here, we used the model-parameters derived from the control subjects. Consistent with the results reported previously (Fig. 4.5B), this analysis still showed significant differences between amygdala subjects and controls in encoding of expected rewards in medial PFC at p<0.001.
Controls > amygdala (switch-stay)
**Figure S4.3. Controlling for behavioral differences between amygdala lesion subjects and controls in signals pertaining to behavioral choice.** To further control for the effects of differences in behavioral performance between the amygdala subjects and control subjects on the fMRI data, we restricted our analysis to only those trials in which both amygdala subjects and controls made correct choices (given the underlying contingencies). For this, we modeled separately trials in which subjects’ action of staying with the same choice, or switching choice was correct given the underlying task contingency, from trials in which subjects’ actions were incorrect given the underlying task contingency. In this figure we show the results of a comparison between switch-stay trials in amygdala subjects and controls, similar to that shown in Fig. 4.4C. Even after controlling for behavioral differences, this analysis revealed a similar result to that reported in Fig. 4.4C. That is, amygdala subjects showed significantly reduced activity in posterior lateral orbitofrontal cortex/anterior insula and anterior cingulated cortex on switch-stay trials compared to controls (an effect which was still significant at p<0.001).
Figure S4.4. Multiple axial slices for both amygdala lesion subjects. Multiple axial slices of un-normalized T1-weighted structural images for both amygdala lesion subjects show that the rest of the brain is generally unaffected by the disease. Axial slices marked with an asterisk are shown in Fig. 4.1, in which the amygdala lesions for both subjects are compared to the intact amygdala of a typical control subject. In the asterisk-marked slices, the bilateral calcification of the amygdala due to Urbach-Wiethe disease can be seen as a loss of signal (dark) on these T1-weighted structural MR scans.
Chapter 5

THINKING OF YOU THINKING OF ME

Competing successfully with an intelligent adversary depends on the ability to mentalize the opponent’s state of mind in order to anticipate the opponent’s behavior in the future. Here, we explore the computational underpinnings of such a capacity by scanning human subjects with fMRI while they engaged in a simple strategic game against a real opponent. Subjects were found to employ a sophisticated strategy, whereby they used knowledge of how their actions would influence the actions of their opponent to guide their choices. Moreover, specific computational signals required for the implementation of such a strategy were present in medial prefrontal cortex and superior temporal sulcus, providing insight into the basic computations underlying competitive strategic interactions, and their associated neural bases.

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vi Work done in collaboration with Peter Bossaerts and John P. O’Doherty.
Humans, like many other primates, live in a highly complex social environment in which it is often necessary to interact with, and compete against, other individuals in order to attain reward. Success against an intelligent adversary with competing objectives likely depends on the capacity to infer the opponent’s state of mind, in order to predict what action the opponent is going to select in future and to understand how an individual’s own actions will modify and influence the behavior of one’s opponent. This ability is often referred to as “mentalizing” and has been linked to a number of specific regions thought to be specifically engaged when processing socially relevant stimuli, and especially when inferring the state of mind of others\textsuperscript{145}. Neuroimaging studies in humans have implicated a specific network of brain regions including dorsomedial prefrontal cortex, posterior superior temporal sulcus (STS), and the temporal poles\textsuperscript{146, 147} while subjects engage in tasks relevant to mentalizing, such as evaluating false beliefs or social transgressions\textsuperscript{148, 149}, describing the state of biological movements\textsuperscript{150-152}, and while playing interactive games\textsuperscript{153-155}. However, although these studies have provided insight into what brain regions may be involved in the capacity to mentalize, the question of how this function is implemented at the neural level has received relatively little attention to date.

The goal of the present study was to begin to characterize the basic computational signals underlying the capacity to mentalize, and to link different components of these putative signals to specific brain regions. In order to assess competitive interactions experimentally, we studied pairs of human subjects while they played each other in a two-player strategic game called the “inspection” game (or generalized matching pennies), in which opponents have competing goals (Figure 5.1A and 5.1B). One of the players was being scanned with fMRI, while their opponent was playing outside the scanner. The “employer” could either “inspect” or “not inspect,” while the “employee” could either “work” or “shirk.” The employer received 100 cents if he/she did “not inspect” and the employee “worked,” and received 25 cents if he/she “inspected” and caught the employee “shirking.” Otherwise he/she got zero cents. In contrast, the “employee” got 50 cents for “working” when the employer “inspected,” and for “shirking” when the employer did “not inspect,” otherwise getting zero cents as well. Both players had competing objectives, in that when one player won in a given trial, the other one lost.
A player can in principle use a number of different strategies in order to try to win in such a game. Perhaps the simplest strategy is on each trial to simply choose the action that in the recent past gave the most reward. This strategy is referred to as reinforcement learning (RL), and approximates the optimal solution for many different types of decision problem in non-strategic contexts. However, such a strategy would be devastating for an individual in a competitive scenario, because a clever opponent could detect the regularity in the reinforcement learners’ choices in order to work out what action the reinforcement learner is going to choose next and exploit that knowledge by choosing the confounding response.

A more sophisticated approach is to try to predict the opponent’s next actions by taking into account the history of prior actions by the opponent, and then choosing the best response to that predicted action, a strategy known as “fictitious play.” A fictive learner is, in contrast to a reinforcement learner, employing an elementary form of mentalizing, because they are engaging a representation of the actions and intentions of their opponent.

However, an even more cognitively sophisticated and “Machiavellian” strategy a player could use in this game is to not only track the opponent’s actions, but also incorporate knowledge of how one’s own actions influence the opponent’s strategy. Simply put, this involves a player building a prediction of what the opponent will do in response to the player’s own actions. For example, the more the employer “inspects,” the higher the probability the employee will “work” in subsequent trials. The employer can then use this knowledge to make choices with higher expected rewards in following trials, i.e., “not inspect.” We will term this strategy the “influence” learning model (see Table 5.1 for a comparison of the different models).

To address which of the above strategies most closely captured subjects’ behavior, we fit each model to behavior separately and compared the goodness of fit of each model. We found that the influence learning model provided a significantly better fit to subjects’ behavior (p<0.005 paired t-test) than did either the fictitious play rule or the reinforcement learning.
learning rule, even when taking into account the different number of free parameters in each model by performing an out-of-sample test (Fig. 5.1C; Fig. S5.1). Fig. 5.1D shows the relationship between the probability of an action being selected as predicted by the influence model, and actual subject choices. These findings suggest that subjects are not only using representations of the opponents’ future choices to guide choice, but are also employing representations of the opponents’ likely responses to their own actions.

We next analyzed the fMRI data from the player being scanned, to determine whether we could find evidence of neural signals reflecting the different components of the influence model, and if so, whether those signals are better accounted for by the influence model than by the fictitious play or simple RL models. A comparison of brain signals associated with the expected reward of the chosen action, as predicted by each model, is shown in Fig. 5.2A. Expected value signals from the influence model were significantly correlated with neural activity in medial orbitofrontal cortex (mOFC), medial prefrontal cortex (encompassing both ventral and dorsal aspects, significant at p<0.05 corrected for small volume [SVC]), and right temporal pole (p<0.05 SVC). By contrast, only weak correlations with the expected value signals from the fictitious play model were found in medial orbitofrontal cortex, whereas no significant correlations were found with expected value as computed by the simple RL model. We next tested for brain regions showing a significantly better fit to the influence model than the RL model. This analysis revealed significant effects extending from mid to dorsal medial prefrontal cortex (p<0.05 SVC; Fig. 5.2B), as well as in the right temporal pole (Fig. S5.2). The regression fits of the three models are shown in Fig. 5.2C for medial prefrontal cortex, demonstrating the superiority of the influence model in accounting for neural activity in this area. We then binned BOLD activity from mPFC according to the expected reward as predicted by the influence model, to illustrate the relationship between evoked fMRI responses and the model predictions (Fig. 5.2D). These data show that the influence model provides a significantly better account of the neural data in medial prefrontal cortex than does a simple RL model.

We next set out to differentiate between the effects of the influence model and the more closely related fictitious play model in this area. For this, we looked specifically at the
points in the experiment when the predictions of these two models differ. In particular, the influence model predicts that the expected value following a switch in action choice (i.e., moving from working to shirking or vice-versa on successive trials) is on average higher than the expected reward when not switching choice (i.e., taking the same action on successive trials), whereas the fictitious play and indeed RL models predict exactly the opposite (Fig. 5.2E). This effect is greatest for the employee, as behavioral fits indicate that subjects exert more influence on their opponent when playing this role. An analysis of BOLD activity in the medial prefrontal cortex region of interest at the time of choice, revealed a positive signal in this area on switch compared to non-switch trials for the employee, consistent with the predictions of the influence model, but not with either the fictitious play or simple RL models (Fig. 5.2F). These results therefore suggest that the influence model does indeed account better for neural activity in medial prefrontal cortex than the fictitious play model.

At the time of outcome, according to the influence model, a player needs to update his/her expectations of the opponent’s strategy using two different components: an influence update signal found only in the influence model and not in either of the other two models, which encodes the magnitude by which the opponent’s behavior adapts due to a player’s own action; and, in common with both the RL model and the fictitious play model, a prediction error signal that encodes the discrepancy between expected and actual rewards. We found that neural activity in another key component of the mentalizing network, superior temporal sulcus (bilaterally), was significantly correlated with the influence update signal (p<0.05 SVC; Fig. 5.3A), suggestive of a role for this region in guiding the update of expected value representations in medial PFC. Prediction error signals were found to correlate with neural activity in ventral striatum bilaterally (see Fig. S5.3), consistent with many previous findings implicating this area in prediction error coding. Moreover, this analysis revealed significant prediction error effects in medial PFC, suggesting that this signal could also contribute to the updating of expectations in this region.

To further investigate differences between the influence and fictitious play models, we examined between-subject variability in the degree to which the influence model provided
a better fit to subjects’ behavior than the fictitious play model, by comparing the difference in the likelihoods of the two models, and correlating that with neural activity elicited by the influence update signal. This measure can be taken as an assay of the individual differences in the degree of influence-based strategizing within our subject group. We found a significant between-subject correlation in the degree of influence activity and the difference in likelihoods between the influence and fictitious play models in dorsomedial prefrontal cortex (p<0.05 SVC; Fig. 5.3B). These results suggest that the more subjects strategize, the stronger the influence-based model correlates with neural activity in this region.

Most studies involving mentalizing elicit activity in mPFC\textsuperscript{145-147}. In this paper, we show that mPFC not only computes expected rewards of the selected choice in a two-person game, but is specifically recruited in the sophisticated update of the opponent’s state estimate (strategy) through prediction error and influence signals that also elicit BOLD activity in this region. Furthermore, another component of the “mentalizing” network, posterior STS, was found to be correlated with the influence a player’s action had on the opponent’s strategy. This area has previously been implicated in processing stimuli related to living agents and biologically relevant motion\textsuperscript{150, 158}. Here, we provide evidence for a computationally specific role of this region in encoding influence signals during strategic social interactions.

In this study we have taken the first steps in attempting to characterize the neural underpinnings of mentalizing during strategic interactions in precise computational terms. We have shown that a computational model which captures a sophisticated strategy in which individuals keep track not only of the actions of the opponent, but of how opponents are influenced in response to their own actions, provides a good account of behavior during performance of a simple strategic game. We have also shown that specific computational signals needed for the implementation of such a strategy are encoded in specific brain regions, such as dorsomedial prefrontal cortex and posterior superior temporal sulcus. These areas have previously been implicated in mentalizing and in “theory of mind,” but have never before been shown to encode specific computational signals that may potentially underlie such capacities. While in the present study players understand the
effects of influencing the opponent, an open question is how they could use that knowledge in order to alter the opponent’s behavior so as to receive bigger future rewards, by means such as reputation building and teaching\textsuperscript{159}, or Stackelberg strategies (in which one player commits to a certain strategy and forces the other player to follow suit\textsuperscript{160}). Furthermore, although in the present study human players always faced real human opponents, an interesting question for further study would be whether similar mechanisms are engaged in these areas when subjects are playing an intelligently adaptive but non-human computer, a manipulation often used when probing “theory of mind” areas in human imaging studies\textsuperscript{153-155}. Another open question is whether other animals besides humans have the capacity for sophisticated strategic computations of this sort, or whether the capacity to engage in such high level strategies is a uniquely human trait. Although previous studies of strategic game playing in rhesus macaques indicate that these animals do use simple RL and possibly fictitious updating\textsuperscript{161-163}, it has not yet been addressed whether they are capable of higher-level strategizing as found here in our human subjects. More generally, the present results show how the application of quantitative computational models to neuroimaging and behavioral data can be used to advance knowledge not only of simple learning situations, but also to unlock the complexities of social and strategic interactions\textsuperscript{22,164}. 
**A** Task

Trial start, choice presented for ~1500ms

-500ms Players make choice

**B** Game payoff

<table>
<thead>
<tr>
<th>Employee</th>
<th>Employer</th>
</tr>
</thead>
<tbody>
<tr>
<td>not inspect</td>
<td>not inspect</td>
</tr>
<tr>
<td>work</td>
<td>100¢</td>
</tr>
<tr>
<td>0¢</td>
<td>50¢</td>
</tr>
</tbody>
</table>

**C** Model likelihood

<table>
<thead>
<tr>
<th>Model</th>
<th>Log Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>RL</td>
<td>-68</td>
</tr>
<tr>
<td>Fictitious</td>
<td>-67</td>
</tr>
<tr>
<td>Influence</td>
<td>-68</td>
</tr>
</tbody>
</table>

**D** Subject choices

<table>
<thead>
<tr>
<th>Subject Choice</th>
<th>Model Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>0.6</td>
<td>0.7</td>
</tr>
</tbody>
</table>

8.5 seconds - Trial ends

5 seconds - outcome of players’ choices, shown for 1500ms
**Figure 5.1. Inspection game and behavioral results.** (A) Two interacting players are each individually given two action choices at the beginning of each trial. Players are given one second to respond, and their choices are highlighted with a red frame for another second before being covered with a blank screen. Five seconds after the start of the trial, the actions of both players are shown to each player for 1.5 seconds, with the payoff each one individually receives shown at the top. (B) Payoff matrix for the inspection game used in this paper. (C) Log likelihood errors for each computational model tested shows that the influence model, which incorporates the effects of players’ actions influencing their opponents, has a better fit to subjects’ behavior than the RL and fictitious play models (to account for overfitting and the effects of differences in free parameters between models we also computed out-of-sample log likelihoods, which still yielded the same results, as shown in Fig. S5.1). (D) Furthermore, the actual probability of a player taking a specific behavioral action is linear with respect to the probability of choosing that action as computed by the influence model. Here, behavior and predictions are shown separately for the “employer” and “employee.”
A Model expected reward

B Influence > RL

C Model comparison

D Expectation in mPFC

E Model Predictions Switch vs. No-Switch

F Brain Switch vs. No-Switch
Figure 5.2. Expected reward signals. (A) At the time of choice, the expected reward of the action selected by a player is shown across the brain, as calculated by different computational models. The expected reward signal from the influence model is correlated significantly with BOLD responses in medial OFC (0, 36, -21mm, z=3.56), medial PFC (-3, 63, 15mm, z=3.29), and right temporal pole (42, 15, -39, z=3.98) – the latter two areas surviving at p<0.05 correction for small volume (SVC) within an 8mm sphere centered on co-ordinates from areas implicated in mentalizing\textsuperscript{147}, while only the fictitious play model has significant activity in mOFC (at p<0.001). The RL model had no significant activity correlating with expected reward anywhere in the brain. (B) An analysis to test for areas showing neural activity related to expected reward that is explained significantly better by the influence model than by the RL model revealed statistically significant effects in mPFC (-3, 57, 12mm, z=3.11; p<0.05 SVC). Panel (C) shows the average correlation coefficients for each model from the area reported in (B) (extracted from all voxels showing effects at p<0.005 in mPFC). (D) fMRI activity in mPFC shows a linear relation with binned expected reward probabilities as computed by the influence model (fMRI activity extracted from individual peaks in a 10mm search radius centered on the peak from panel B). (E) The computational models tested in this paper make distinctly different predictions of the overall expected reward signals after switching actions (“switch”), or sticking to the same action (“non-switch”), as a consequence of influencing the opponent. Intuitively, the underlying reason is that both RL and fictitious play will most likely “stay” after a reward, and “switch” after a non-reward. However, the influence model has a higher incentive to “switch”, even after receiving a reward. That is, expected reward signals associated with a specific action do not necessarily increase after the receipt of a reward when taking into consideration the influence that specific action exerts on the opponent’s strategy. (F) fMRI responses in mPFC at the time of choice on switch compared to non-switch trials show a response profile consistent with the influence model and not the fictitious play models or RL models (the data is extracted from a 10mm sphere centered on the peak from panel B). The difference between the employee and employer was significant at p=0.02.
A Influence update

B Influence update modulated by Likelihood diff.
Figure 5.3. Influence signals in the brain. (A) At the time of outcome, the influence update of the inferred opponent’s strategy shows significant correlations with activity in STS bilaterally (-57, -54, 0mm, z=3.32 and 60, -54, 9mm, z=3.35, p<0.05 SVC). (B) The degree to which a subject thinks he/she is influencing his/her opponent can be measured by taking the difference in log-likelihood fits between the influence and fictitious models on each player’s behavior. Likewise, brain regions invoked in computing the influence on the opponent will correlate more strongly with the influence model for subjects invoking this approach, when compared to subjects that do not. Influence signals were found to significantly co-vary with the model likelihood difference (influence-fictitious) across subjects in mPFC (-3, 51, 24mm, z=4.09; p<0.05 SVC). The right panel shows the relationship between influence regression coefficients and model likelihood differences in mPFC.
Table 5.1. Model update rules. The RL model updates the value of the chosen action with a simple Rescorla-Wagner prediction error $\delta_i$, as the difference between received rewards and expected rewards, where $\eta$ is the learning rate. The fictitious play model instead updates the state (strategy) of the opponent $p_i^*$ with a prediction error $\delta_i^p$ between the opponent’s action and expected strategy. The influence model extends this approach by also including the influence $\lambda_i^p$ a player’s own action has on the opponent’s strategy. For more details, see Materials and Methods in this chapter.
MATERIALS AND METHODS

Subjects
Thirty-two healthy, normal subjects participated in this study, of which sixteen (25±1 years old, 7 female) were scanned while playing a competitive game in pairs with the other sixteen. Subject pairs were pre-screened to make sure that the subjects in each pair did not know each other before the experiment in order to reduce the possibility of collusion. However, one pair of subjects was in fact discarded due to evidence of collusion during the game. The subjects were also pre-assessed to exclude those with a prior history of neurological or psychiatric illness. All subjects gave informed consent and the study was approved by the Institute Review Board at Caltech.

Task description
Subjects participated in pairs in a simple interaction game: the inspection game (or generalized matching pennies game). The inspection game involves the interaction of two players: one with the role of employer and the other with the role of employee (Fig. 5.1A). On each trial, each player is presented with two choices. The subject whose role is the employer can choose to either “inspect” or “not inspect,” while the subject whose role is the employee can choose to either “work” or “shirk.” After players have individually made their choice, they are both shown the choice of their opponent, and rewarded according to the payoff matrix illustrated in Fig. 5.1B. Players have competing incentives, in that when one player is rewarded, the other player is not. Thus, both players will seek to maximize their rewards by trying to outguess the opponent’s next move (offensive play), while simultaneously making their own move as unpredictable as possible (defensive play). The payoff matrix’s association with players’ roles and actions remained fixed throughout an experiment, but was rotated across experiments to create a balanced design with respect to roles and actions. However, for convenience, in this paper we will always refer to the “employer” as having a payoff with action “not inspect” delivering a high reward (100 cents) and action “inspect” a low reward (25 cents) – contingent on the opponent’s play; and will refer to the ‘employee’ as having a payoff with both actions always delivering the same reward (50 cents) – contingent on the opponent’s play.
Each pair of subjects underwent three game sessions. One subject used a computer terminal and keyboard to play the game, while the other was in the scanner using goggles as visual input (Resonance Technologies, Northridge, CA, USA), and a button box to choose an action. The first session was for training, while the second two are reported in this paper. Player roles alternated between the two subjects in each session. Thus, the scanned subjects reported in this paper played both roles in subsequent sessions (employer and employee). In addition to the game trials, we also included randomly intermixed null event trials which accounted for 33% of the total number of trials in a session. These trials consist of the presentation of a fixation cross for 7 seconds. Before entering the scanner, subjects were informed that they would receive what they earned in a randomly selected session, plus an additional $10 dollars.

**Reinforcement Learning**

Reinforcement learning (RL) is concerned with learning predictions of the future reward that will be obtained from being in a particular state of the world or performing a particular action. In this paper we use a simple RL model in which action values are updated via a Rescorla-Wagner (RW) rule\(^\text{14}\). On a trial \(t\) in which action \(a\) is selected, the value of action \(a\) is updated via a prediction error \(\delta\):

\[
V_{t+1}^a = V_t^a + \eta \delta_t, \tag{5.1}
\]

where \(\eta\) is the learning rate. The prediction error \(\delta_t\) is calculated by comparing the actual reward received \(R_t\) after choosing action \(a\) with the expected reward for that action:

\[
\delta_t = R_t - V_t^a. \tag{5.2}
\]

When choosing between two different states (\(a\) and \(b\)), the model compares the expected values to select which will give it the most reward in the future. The probability of choosing action \(a\) is:
\[ p^a = f(V^a - V^b), \]  
\[ \text{(5.3)} \]

where \( f(z) = 1/(1 + e^{-\beta z}) \) is the Luce choice rule\(^{18}\) or logistic sigmoid, and \( \beta \) reflects the degree of stochasticity in making the choice (i.e., the exploration/exploitation parameter).

**Fictitious Play**

In game theory, a first-order fictitious play model\(^{156}\) is one in which a player infers the probability that the opponent will choose one action or another, and then decides so as to maximize the action’s consequent expected reward. The opponent’s probability \( p^* \) of choosing an action \( a' \) is dynamically inferred by tracking the history of actions the opponent makes:

\[ p_{t+1}^* = p_t^* + \eta \delta_t^p, \]  
\[ \text{(5.4)} \]

where \( \delta_t^p = P_t - p_t^* \) is the prediction error between the opponent’s expected action \( p^* \) and whether the opponent chose action \( a' \) at time \( t \) (\( P = 1 \)), or chose another action (\( P = 0 \)). Given the opponent’s action probabilities \( p^* \), the expected value for each of the player’s actions can be calculated using the payoff matrix of the game. A stochastic choice probability can then be calculated using equation 5.3. For the inspection game described in this paper, this can be summarized as follows: calling \( p \) the probability that the “employee” will “work,” and \( q \) the probability that the “employer” will “not inspect,” and using the payoff matrix of the game (Fig. 5.1B – in the following formulations, payoffs were expressed in 25 cent units for convenience), the decision of each player is:

\[ p = f(2 - 4q^*) \]  
\[ q = f(5p^* - 1), \]  
\[ \text{(5.5)} \]

where \( q^* \) and \( p^* \) are the inferred probabilities of the opponent’s actions estimated using equation 5.4.
An equivalent formulation of fictitious play is one in which the values of actions are learned directly as in reinforcement models, instead of tracking the opponent player’s action probability. For this, not only the value of the chosen action is updated with the reward that was received (as in equation 5.1), but also all other actions are penalized proportional to their foregone rewards\textsuperscript{143, 144}. Either approach posits knowledge of the structure of the game to update the variable estimates and arrive at a correct expected value for the actions of each player.

**Influencing the Opponent**

How much does a player’s next decision change given the action of the opponent? Replacing the update of the inferred opponent’s strategy (equation 5.4) in a player’s decision (equation 5.5), and Taylor expanding (around $\eta=0$):

\begin{align*}
\Delta p & \approx -\eta 4 \beta p_i (1 - p_i)(Q_i - q_i^*) \\
\Delta q & \approx +\eta 5 \beta q_i (1 - q_i)(P_i - p_i^*) .
\end{align*}

The sign difference in both terms is determined by the competitive structure of the game, namely, that the employer wants to “inspect” when the employee “shirks,” while the employee wants to “shirk” when the employer does “not inspect.” A player can obtain a more accurate inference of the opponent’s action strategy by incorporating the influence his/her own action has on the opponent. Thus, at the end of each trial both players update the estimates of their opponent such that:

\begin{align*}
p_{i+1}^* & = p_i^* + \eta_1 (P_i - p_i^*) - \eta_2 4 \beta p_i^* (1 - p_i^*)(Q_i - q_i^{**}) \\
q_{i+1}^* & = q_i^* + \eta_1 (Q_i - q_i^*) + \eta_2 5 \beta q_i^* (1 - q_i^*)(P_i - p_i^{**}) ,
\end{align*}

where $q^{**}$ and $p^{**}$ are the inferred probabilities that the opponent has of the player itself (second-order beliefs). Thus, this gives two clear signals: the prediction error as the first term and the influence update as the second term. The influence update, or how much a player influences his/her opponent, is proportional to the difference between the action a
player took and what the opponent thought was the player’s strategy. These second-order beliefs can be inferred by the player directly from the inferred opponent’s strategy by inverting equation 5.5:

\[
p_i^* = \frac{4}{5} - \frac{1}{5\beta} \log \left( \frac{1 - q_i^*}{q_i^*} \right)
\]

\[
q_i^* = \frac{1}{2} + \frac{1}{4\beta} \log \left( \frac{1 - p_i^*}{p_i^*} \right).
\]  

(5.8)

**Behavioral Data Analysis**

Model parameters were estimated by maximizing the logistic log likelihood of the predicted decision probabilities generated by a model against the actual behavior of all subjects. Parameters for the employer and employee roles were fitted separately to account for any differences. We used the multivariate constrained minimization function (fmincon) of the Optimization Toolbox 2.2 in Matlab 6.5 (www.mathworks.com) for this fitting procedure. All behavioral data shown corresponds to the actions of all 30 participants.

**fMRI data acquisition**

Functional imaging was conducted using a Siemens 3.0 Tesla Trio MRI scanner to acquire gradient echo T2* weighted echo-planar (EPI) images. To optimize functional sensitivity in OFC we acquired the data using an oblique orientation of 30° to the AC-PC line. A total of 580 volumes (19 minutes) were collected during the experiment in an interleaved-ascending manner. The imaging parameters were: echo time, 30ms; field-of-view, 192mm; in-plane resolution and slice thickness, 3mm; TR, 2 seconds. High-resolution T1-weighted structural scans (1x1x1mm) were acquired for anatomical localization. Image analysis was performed using SPM2 (Wellcome Department of Imaging Neuroscience, Institute of Neurology, London, UK). Pre-processing included slice timing correction (centered at TR/2), motion correction, spatial normalization to a standard T2* template with a resampled voxel size of 3mm, and spatial smoothing using an 8mm Gaussian kernel.
Intensity normalization and high-pass temporal filtering (128 secs) were also applied to the data.96

fMRI data analysis
The event-related fMRI data was analyzed by constructing sets of delta (stick) functions at the time of the choice, and at the time of the outcome. Additional regressors were constructed by using the model estimated choice expected values as a modulating parameter at the time of choice; and one or more (depending on the model) update signals as modulating parameters at the time of outcome. All of these regressors were convolved with a canonical hemodynamic response function (hrf). In addition, the 6 scan-to-scan motion parameters produced during realignment were included to account for residual motion effects. These were fitted to each subject individually, and the regression parameters were then taken to the random effects level. All reported fMRI statistics and uncorrected p-values arise from group random effects analyses (N=15), and small volume corrected p-values are obtained from 8mm spheres around regions of interest previously implicated in mentalizing from a meta-analysis by Frith and Frith147. Specifically, coordinates were defined from the center of each of the areas delineated from the meta-analysis: mPFC (0, 56, 19mm), STS (±53, -51, 10mm), and temporal poles (±46, 11, -35mm). An across-subject model likelihood difference modulator was also fitted at the second level, alongside the mean group-level effect, when studying the influence signal in Fig. 5.3.

We also compared the influence and RL algorithms to each other by fitting both models at the same time with the fMRI data (Fig. 5.2B). To make both models as similar as possible, we normalized all modulating regressors before fitting to the fMRI data. Areas showing significant activity for any regressor of a given model indicated regions that were better explained by that model in comparison to the other.
**Figure S5.1. Out-of-sample model log likelihood.** The out-of-sample model log likelihood controls for models having different number of free parameters when fitting to behavioral data. Models were trained with the first 70 trials for each subject, and then tested on the last 30 trials to obtain an out-of-sample log likelihood. The influence model accounts for subjects’ behavior the best, with an out-of-sample log likelihood of 0.674±0.004, followed by the fictitious play model with 0.685±0.003, and the RL model with 0.687±0.003.
Influence > RL in Temporal Pole and STS
Figure S5.2. Model comparisons with respect to the processing of expected reward signals in the brain. The influence model expected reward signals that are not explained by (orthogonal to) the RL model expected reward signals also activate the right STS, including the right temporal pole at p<0.001 uncorrected.
Influence model prediction error

P < .00001
P < .00005
P < .0001
Figure S5.3. Prediction error signals. The prediction error signals generated by the influence model were correlated with activity in the ventral Striatum bilaterally (9, 6, -18mm, z=4.97; -9, 9, -18mm, z=4.73; both p<0.05 whole brain corrected), mPFC (-9, 57, 6mm, z=4.35), and paracingulate cortex (12, 36, 18mm, z=4.62). This lends support to the suggestion that mPFC is not only involved in calculating expected reward signals derived from inference of the opponent’s game strategy (Fig. 5.2B), but is also involved in the update of the inferred opponent’s strategy through prediction errors (this figure) and influence updates (Fig. 5.3B).
REFERENCES


Bayesian models not only care about the relation between variables in a model, but also about the noise processes in those relations. When some variables in a model can be measured, the Bayesian model can be used to make optimal estimates of the values of the unknown (or hidden) variables. In this short introduction to Bayesian Inference we will go over the basics of this procedure. Other aspects of Bayesian models, such as learning, are left as references at the end.

Conditional Probabilities
In most instances, the relation between two variables \((x, y)\) can be expressed as a function linking them:

\[ y = f(x). \quad (A.1) \]

However, we are not only interested on the dependence between variables, but also on the random noise governing this relation. Instead of assigning a unique value \(y\) given \(x\), we can define a conditional probability of \(y\) given \(x\), or \(p(y/x)\). Thus, depending on the value of \(x\) we do not have a unique value for \(y\), but a probability for every value \(y\) can have. Graphically, the conditional probability relation of random variable \(y\) given \(x\) can be illustrated as follows:
In another terminology, the conditional probability describes how \( x \) generates \( y \), because we know the value of \( x \) (shaded circle) and can describe the probability distribution that \( y \) will have. In many situations, conditional probabilities are modeled assuming Gaussian noise (Fig. A below); but far more fancy relations, such as the multimodal distribution in Fig. B below, could arise in complex real-world situations.

**Figure A.1.** Conditional probability distributions of \( y \), given a fixed value of the random variable \( x \). (A) Given \( x \), \( y \) has a Gaussian probability distribution centered around the most likely value of \( y \) to be obtained given \( x \). (B) \( y \) has a multimodal probability distribution, and the mean of the distribution is not the most likely value of \( y \) to be obtained given \( x \).
Bayes’ Rule

We defined the conditional probability of $y$ given $x$ as a better description of the link between both variables. If we also know the probability distribution of $x$ on its own (priors), we can express the joint probability of both random variables as the multiplication of the probability of obtaining a value $x$ times the probability of obtaining a value $y$ given $x$:

$$p(y, x) = p(y/x)p(x).$$ \hfill (A.2)

The joint probability is the complete description of the relation between random variables. From this expression we can derive the inverse conditional probability, that of $x$ given $y$, through what is known as Bayes’ rule:

$$p(x/y) = \frac{p(y/x)p(x)}{p(y)}, \quad \text{where } p(y) = \int_s p(y/x)p(x)dx.$$ \hfill (A.3)

This is illustrated in a graphical model as follows:

The arrow indicates that we know how the random variable $y$ is generated given $x$. However, if we observe variable $y$ (shaded circle), we would like to be able to infer the value of $x$ given our Bayesian model. The answer is found with Bayes’ rule in equation A.3, which indicates how to calculate the conditional probability (posterior) of $x$ given our observed variable $y$. In most cases, this distribution is unimodal and one can express the
mean of the distribution as the value estimate, and the variance of the distribution as the
error of the inference.

**Simple Examples**

*Dart Players (binary to real random variables)*

Imagine two dart throwers, one quite good and one not so. To measure exactly how good
each one is, a sample of their throwing capabilities can be recorded and a probability
distribution derived. The good thrower, being always close to the mark, would have a tight
probability distribution (blue line), whereas the bad thrower not only has a more dispersed
distribution, but in this case has a systematic bias of hitting to the right of the center target.

Thus, given the current player, we know the probability distribution of the location their
darts will hit. These are Gaussian distributions with a certain mean and variance. We can
now also answer the inverse question. Given the location of a recently thrown dart, can we
infer which player threw that dart? Returning to our descriptions of random variables, the
location the dart reaches is a continuous random variable $y$ that is generated by a binary
random variable $x$ which can take two states, either the good player (g) was playing, or the
bad player (b) was playing. Using equation A.3, the posterior probability of the good player
(g) having thrown the dart can be calculated as:

$$p(x = g / y) = \frac{p(y / x = g)p(x = g)}{p(y / x = g)p(x = g) + p(y / x = b)p(x = b)}$$  \hspace{1cm} (A.4)
where $p(y/x = g)$ is the Gaussian distribution of dart locations given that the good player played, and $p(y/x = b)$ the probability distribution of dart locations given that the bad player played. The prior probabilities $p(x = g)$ and $p(x = b)$ indicate knowledge of how likely it was that either player was playing before the dart location was measured. For example, the good player probably plays more often, and thus $p(x = g) = 0.75$ while $p(x = b) = 0.25$. In general, if no assumptions can be made about how often each player plays, all priors are left equal and the evidence (dart location) is left to decide who is the most probable player to have thrown the dart.

**Burglar and Alarm (binary to binary random variables)**

Binary random variables can predict other binary random variables as in this case. The random variable $x$ consists of two states: a burglar is entering the house (b), or nothing is happening (n). How do these states influence the binary variable $y$: whether an installed alarm system will go off (a) or not (na). How $x$ generates $y$ is depicted in the following diagram.
If a burglar is present, the alarm will go off with 0.9 probability, but might not go off at all (0.1 probability). On the other hand, if nothing is happening the alarm will not go off (0.8 probability), but there still is a chance of the alarm going off due to other accidental circumstances (0.2 probability). These conditional probabilities can be written down as a matrix table:

\[
p(y/x) = \begin{pmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{pmatrix},
\]

where \( x \) states are columns and \( y \) states are rows. If we have equal priors for burglars being present or not (the priors then drop out from equation A.3), we can infer that the posterior probability that a burglar is present given that the alarm went off is 0.82:

\[
p(x=b/y=a) = \frac{p(y=a/x=b)}{p(y=a/x=b) + p(y=a/x=n)} = \frac{0.9}{0.9 + 0.2} = 0.82.
\]

**Complex Graphical Models**

Models can be built that describe the conditional relations among many random variables, where each can be continuous, binary, or of other types:

![Figure A.2. Directed graphical model linking six random variables](image)
The complete model can be expressed as a joint probability distribution \( p(a, b, c, d, e, f) \) linking the random variables. If some of the random variables are observed (shaded nodes), then the hidden variables (not observed) can be inferred by calculating the conditional probability of \( a, c, d \) given \( b, e, f \)

\[
p(a, c, d / b, e, f) = \frac{p(a, b, c, d, e, f)}{\sum_{a, c, d} p(a, b, c, d, e, f)} \tag{A.7}
\]

and setting the values of the observed variables \( p(a, c, d / b = \alpha, e = \beta, f = \lambda) \). If we are only interested in inferring the value of random variable \( d \) given our observation of \( b, e \) and \( f \), we marginalize (integrate) over the unobserved (hidden) random variables we are not interested in:

\[
p(d / b = \alpha, e = \beta, f = \lambda) = \frac{p(a, c, d / b = \alpha, e = \beta, f = \lambda)}{\sum_{a, c, d} p(a, c, d / b = \alpha, e = \beta, f = \lambda)} \tag{A.8}
\]

A directed graphical model (Fig. A.2) represents a joint probability distribution over the model’s random variables that can be factorized into a product of conditional probabilities. This factorization is implied by which random variables are connected with arrows.

\[
p(a, b, c, d, e, f) = p(f / d, e)p(d / c)p(e, c)p(c / a, b)p(a)p(b) \tag{A.9}
\]

In particular, when a random variable is generated by two other random variables (such as \( a \) and \( b \) having arrows pointing at \( c \), the conditional probability \( p(c / a, b) \) depends on both of these variables. Depending on the underlying distribution model, it may also be possible to factorize the conditional probability such that \( p(c / a, b) = p(c / a)p(c / b) \).

Having a factorial joint probability distribution allows for efficient message passing inference methods for calculating conditional probabilities given observed variables, in which each hidden variable is inferred locally by only using information from other random variables it is linked to in the graphical model. A more complete description of these
algorithms is beyond the scope of this short introduction to Bayesian inference, and the reader is referred to excellent literature on the subject\textsuperscript{24,92}. 

**Hidden Markov Model**

An example of a common graphical model is the Hidden Markov Model\textsuperscript{92} illustrated in the following diagram.

Random variable $x$ is a binary random variable, and $y$ is usually a continuous random variable, the indices indicate the passage of time. The conditional probability $p(x_{t} / x_{t-1})$ is usually a fixed probability matrix as in the previous binary to binary example, and the $p(y_{t} / x_{t})$ conditional probability is usually a Gaussian distribution as in the previous binary to continuous example. The most common computational application of Markov models has been in speech recognition. When the hidden random variable $x$ is continuous, then the model is known as a Kalman Filter\textsuperscript{101}.

To infer the value of the $x$ random variable in the last time step, one can use equation A.7 and A.8 to take into account the observed variables and marginalize away the hidden ones. A result due to the factorizable structure of this model, is that the inferred probability distribution (posterior) at time $T-1$ can be used directly when calculating the posterior at time $T$ in the following manner:
\[ p(x_T / ...) = \frac{p(y_T / x_T) \sum_{X_{T-1}} p(x_T / x_{T-1}) p(x_{T-1} / ...)}{\sum_{X_T} p(y_T / x_T) \sum_{X_{T-1}} p(x_T / x_{T-1}) p(x_{T-1} / ...)} . \] (A.10)

This is the idea behind the message-passing algorithm. Instead of marginalizing over the whole joint probability distribution, local posteriors can be calculated and then used further along (the next time step in this case) to calculate the next local posterior distribution. A simpler way to express this in two steps is:

\[
\begin{align*}
Prior(x_t) &= \sum_{X_{t-1}, \text{states}} P(x_t / x_{t-1}) Posterior(x_{t-1}) \quad \text{(A.11a)} \\
Posterior(x_t) &= \frac{P(y_t / x_t) Prior(x_t)}{\sum_{X_t, \text{states}} P(y_t / x_t) Prior(x_t)} . \quad \text{(A.11b)}
\end{align*}
\]

The Markov property of the model is based on the fact that the posterior at times \( t \) can be inferred using only the current evidence and the posterior at time \( t-1 \), and no other information from states further in the past.

**Markov Decision Processes**

Markov models can also be created to understand decision making processes across time. A decision random variable can be added to the model, such that it influences the hidden state \( x \) at every time step (see figure below). In general the decisions are observed, but the model can also be used to infer what decision was made given an observation as described previously. In the literature, these systems are usually referred to as Partially Observed Markov Decision Processes (POMDP). If outcomes \( y \) have a reward associated with them, then future expected rewards can be inferred (before being observed) given a decision to be made. It is this model that is explained in more detail in Chapter 2.
Learning Graphical Models

In this brief introduction to inference in Bayesian models we have not embarked on the topic of learning the model. In general, model parameters (such as means and variance for Gaussian conditional distributions, or probabilities associated with binary conditional distributions) can be treated as random variables themselves, so that learning is equivalent to inferring their values with the procedures just described. However, in general, more practical methods are used to learn model parameters, and we direct the reader to excellent literature on the subject\textsuperscript{24}. 

![Graphical Model Diagram](image-url)
Appendix B

INFEREN CE DYNAMICAL EQUIVALENTS

In Appendix A Bayesian models were briefly introduced, and once a model is known, how to infer the probability distributions of hidden variables was derived. These are exact solutions that can be calculated given the evidence (or observed variables) and the model parameters. For a hidden Markov model, the inference of the hidden states is calculated with equation A.11. However, approximate dynamical equations that converge to this optimal solution can be derived. The motivation to do this is that these dynamical equations might approximate better the calculations that are carried out at the neural level. Furthermore, a similarity between reward learning, and inference updating will be drawn.

The dynamical equations for a concrete case will be drawn, as an example of the process involved. For this, I will define the HMM used in Appendix A (A.11), and use parameters akin to the reversal learning task used in Chapter 2. For simplicity, I will not include the effect of choice in this derivation. The model parameters I will use are:

\[
P(X_i/X_{i-1}) = \begin{pmatrix} 1-\delta & \delta \\ \delta & 1-\delta \end{pmatrix}
\]  \hspace{1cm} (B.1a)

\[
P(Y_i/X_i) = \begin{pmatrix} \frac{1}{2} + \eta \nu & \frac{1}{2} - \eta \\ \frac{1}{2} - \eta \nu & \frac{1}{2} + \eta \end{pmatrix}.
\]  \hspace{1cm} (B.1b)

The first conditional probability describes how states predict the states in the next time step, where \( \delta \) is the reversal probability. The second conditional indicates the probability of getting a reward or punishment depending on what state I am in (the correct or incorrect state). For example, in the reversal task in chapter 2, \( \eta = 0.1 \) and \( \nu = 2 \), which leaves a conditional probability of getting a reward or punishment of:
\[
P(Y_t/X_t) = \begin{pmatrix} .7 & .4 \\ .3 & .6 \end{pmatrix}.
\]

That is, if I am in the correct state (left column), the probability of getting a reward is .7 (top row), and the probability of getting a punishment .3 (bottom row). Likewise, if I am in the incorrect state (right column), the probability of getting a reward is .4, and the probability of getting a punishment is .6. Thus, the generalized formulation of equation B.1b is such that \(\eta > 0\) indicates how much more reward than punishment is received in the correct state (and more punishment than reward in the incorrect state), and \(\nu > 0\) is an asymmetry factor. From B.1b, the expected value of the correct state is \(E_{\text{correct}} = 2\eta\nu\), and the expected value of the incorrect state is \(E_{\text{incorrect}} = -2\eta\).

**Correct state update**

A dynamical update equivalent of the inference equation A.11 can be derived for model B.1 by Taylor expanding around \(\delta \approx 0\) and \(\eta \approx 0\). Defining \(p\) as the probability of choosing the correct state, A.11 becomes:

\[
p = p + \delta(\frac{1}{2} - p)
\]

where \(P = 1\) if a reward is received, and \(P = 0\) if a punishment is received. Given that there is a higher probability of getting a reward when in the correct state, the receipt of a reward always indicates a bigger probability that one actually is in the correct state, and thus the inferred probability of being in the correct state is increased in B.2b. Conversely, B.2a indicates that the bigger the reversal probability, the faster the probability of choosing the correct state goes to .5 (that is, where the probability of being in either state is undetermined).
Value equivalents

The dynamic update equations defined in B.2b can be converted to value updates. The value update of both choices (the one chosen and the one foregone) becomes:

**When rewarded**

\[ V_{\text{chosen}} = V_{\text{chosen}} + \eta (E_{\text{correct}} - V_{\text{chosen}}) \]

\[ V_{\text{foregone}} = V_{\text{foregone}} + \eta (E_{\text{incorrect}} - V_{\text{foregone}}). \]  

(B.3)

**When punished**

\[ V_{\text{chosen}} = V_{\text{chosen}} + \eta (E_{\text{incorrect}} - V_{\text{chosen}}) \]

\[ V_{\text{foregone}} = V_{\text{foregone}} + \eta (E_{\text{correct}} - V_{\text{foregone}}). \]  

(B.4)

Thus, not only the chosen action is updated, but also the foregone action. Furthermore, the values are not updated with the reward received, and the rewards that could have been received had the other action been taken (as proposed in fictive updating\textsuperscript{143, 144}); but with the expected rewards of the state the outcome is providing evidence for.

To conclude, value update equivalents incorporate the structure of the model when updating the expected value of all choices. Moreover, these are just a proxy for the correct underlying interpretation: that of the inference of hidden state variables.