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# **Force-Detected Nuclear Magnetic Resonance Independent of Field Gradients**

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## ***Abstract***

This thesis describes a new method of magnetic resonance detection based on mechanical displacements caused by magnetic forces, which is general with respect to sample and pulse sequence. A spin-bearing sample placed inside a flexible magnet assembly distorts that assembly in proportion to the sample's magnetization. Radio-frequency fields that modulate the sample's spin magnetization at this detector's mechanical resonance frequency encode magnetic resonance spectra into the detector's trajectory. A key insight is that such mechanical detection can be performed within optimized detectors with no need for field gradients inside the sample volume, circumventing the deleterious consequences of such gradients for sensitivity and resolution. The new method is called Better Observation of Magnetization, Enhanced Resolution, and No Gradient (BOOMERANG), and its sensitivity is predicted to exceed that of inductive detection at microscopic size scales.

A prototype BOOMERANG spectrometer optimized for 3 mm diameter liquid and solid samples is described. The device uses direct digital synthesis of radio-frequency waveforms in its operation and fiber-optic interferometry to detect picometer-scale motions of a detector magnet. This magnet is bound to a tuned mechanical oscillator inside a magnet assembly designed for homogeneity of the magnetic field in the sample. Several types of time-domain FT-NMR spectra on test samples are presented. The data confirm theory and design principles.

The favorable scaling of BOOMERANG's sensitivity and the numerous potential uses for NMR at reduced size scales motivate construction of spectrometers optimized for microscopic samples. Geometric concerns in scaling down BOOMERANG are addressed quantitatively. At size scales where the number of spins is such that mean magnetization is smaller than fluctuations, such fluctuations, if not accounted for, can dominate the noise regardless of the physical detection method used. A measurement paradigm using correlations of these fluctuations to encode spectra is proposed to suppress this quantum noise, and the sensitivity of this method, which we call Correlated Observations Narrow Quantum Uncertainty, Enhancing Spectroscopic Transients (CONQUEST), is analyzed. BOOMERANG and CONQUEST promise to extend the applicability of nuclear magnetic resonance (NMR) for chemical analysis to samples and problems that are currently inaccessible by NMR due to poor sensitivity.

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