Chapter 4

Physical Model

The impedance pump functions based on wave reflections. Each time the tube is compressed, a pair of traveling pressure waves emerges from the ends of the pincher. When the waves reach a mismatch in impedance they will partially reflect. Reflections can occur at the ends of the tube or at the compression site if it is in the closed position when a reflected wave arrives. The sum interaction of these waves is responsible for the build-up of pressure across the pump. The amount of energy imparted into the fluid will depend on the size and shape of the compression as well as the amplitude of compression. The compression profile in time, both its frequency and waveform, will determine the waves generated in the system. And, the mechanical properties of the system such as the diameter, length, materials, pressure, and fluid will ultimately be responsible for the wave speed, attenuation, and the reflectance coefficient in the system.

4.1 Position of Compression

When looking at the data set in figure 3.16, a clear pattern emerges in the net flow versus frequency response of the system. The curves appear similar though at different scales. Two sets of peaks can be determined for the data set. By plotting the frequency of compression versus the position of compression of these two sets we arrive at figure 4.1(a). A similar plot can be made for the amplitude of the flow rate at those peaks. We find that the frequency of the peaks lies on a parabola, while the amplitude lies on a cubic with symmetry about the center of the elastic section.

4.2 Dimensional Analysis

Auerbach et al. presented an analytical model for the impedance pump that requires inviscid flow and consists of a short distensible section at which active compression occurs and two rigid sections of unequal length [1]. Additionally, the solution requires the application of a constant pressure head at the boundaries of the pump. From their model arises a non-dimensional number, λ, that is predicted to be constant (table 4.2).

Property	Symbol
pressure difference across length of pump	ΔP
average flow velocity exiting pump	V
offset from center of compression	h
half the length of pump	L

Table 4.1: Variables used in calculating λ

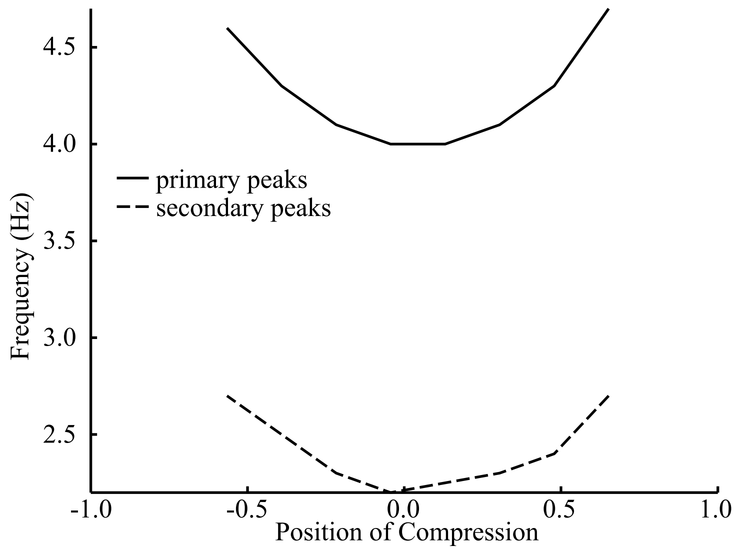
$$\lambda = \frac{\Delta P}{0.5\rho V^2 \frac{h}{L}}$$

Upon application of this number on the data collected, we find that there is no constant line nor trend line found from this formula.

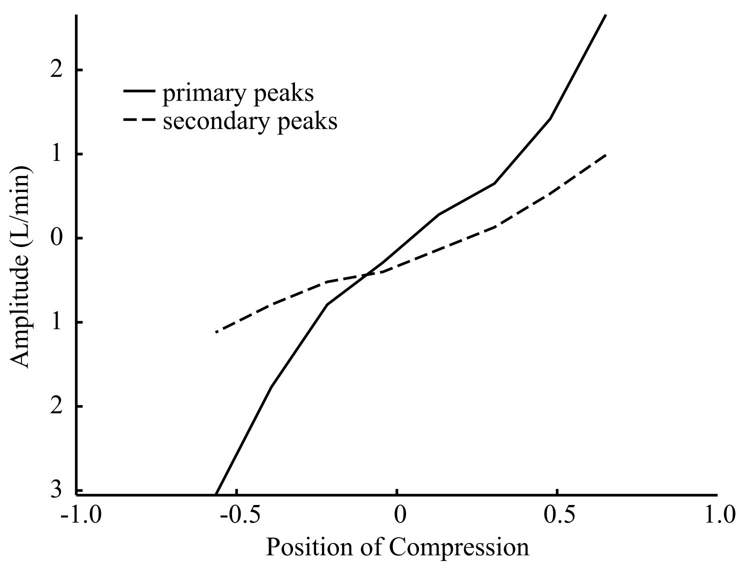
4.3 Lumped Model

Approaches taken thus far to model the behavior of an impedance pump can be divided into two broad categories: lumped models that build an analogy to known solvable systems such as electrical circuits and determine the response of those systems, and computational models that apply known fluid and structure laws to finite cells and determine the response computationally at small temporal and spatial steps. The computational results are more experimental in nature, whereas the lumped models make a prediction of the dominant mechanics.

One such lumped model is presented by Field et al. [3]. This model consists



(a) Peaks in Frequency



(b) Peaks in Amplitude

Figure 4.1: Effect of position of compression on the frequency and amplitude of two distinct peak sets chosen from the data set in figure 3.16. The position of the compression has been centered at and non-dimensionalized by half the length of the elastic section.

of a three-element electrical analog. A resistor represents both the viscous losses and losses due to the convergence and divergence of the cross sectional area. An inductor represents the inertia of the fluid. A capacitor represents the compliance of the surrounding vessel. All the terms are time variant, but not spatially variant. This model then uses a time variant pressure represented by a voltage to actuate the tube. This model has some interesting results: the flow and pressure have a complex periodic oscillation, the net pressure as a function of frequency displays a chaotic response, and period doubling is observed at increasing driving frequencies. However, this type of formulation cannot incorporate the wave propagation that seems to be the underlying cause for the unique frequency responses and resonant behaviors.

4.4 Wave Pulse Model

We offer a third approach to modeling the impedance pump and predicting its behavior. Experimental observation has shown us that wave propagation and reflection on the surface of the pumping element plays an important role in the behavior of the impedance pump. Starting from this point, we can create a wave model designed to mimic the wave properties of the impedance pump.

We begin with a line of fixed length that represents the length of the impedance pump along which a wave can travel.

$$-L \leq x \leq L$$

Compression parameters can be chosen including the location of the center of compression, width, period and duty cycle (table 4.2).

Property	Symbol	Range
compression location	l	$-L < l < L$
width of compression	w	$0 \leq w < L - \|l\|$
period	T	
duty cycle	d	$0 \leq d \leq 1$

Table 4.2: Compression parameters

We use the assumption that a pair of pressure waves are emitted each time the "tube" is compressed. These waves are allowed to travel along the line reflecting any time they reach the ends of the line or the compression location if in the closed position. Additional configurable parameters of the model include the time step between calculations, wave speed, total simulation time, amplitude decay constant, reflectance coefficient, initial wave amplitude, pulse width, and pulse waveform (table 4.3).

Property	Symbol	
time step	dt	
wave speed	c	
total simulation time	sim_t	
amplitude decay constant	r	$0 \leq r \leq \frac{1}{dt}$
reflectance coefficient	R	$0 \leq R \leq 1$
initial wave amplitude	A_0	
pulse width	p	
waveform	$P(x)$	$P(x) = e^{-\frac{2x}{p}^2}$

Table 4.3: Additional model parameters

For each wave pair emitted, their start time, travel time, total distance traveled, directions, amplitudes, and positions are computed for small temporal steps.

- The **start time** for each pair is dependent on the compression period and is an integer multiple of the period.

$$iT$$

- The **travel time** is incremented by dt for each time step above the start time. It is equivalent to

$$= \begin{cases} t - iT & \text{for } t \geq iT; \\ 0 & \text{for } t < iT; \end{cases}$$

- The **total distance traveled** is the wave speed multiplied by the travel time.

$$= \begin{cases} c(t - iT) & \text{for } t \geq iT; \\ 0 & \text{for } t < iT; \end{cases}$$

- The **direction** of each wave changes sign for each reflection encountered.

$$\text{dir} = \begin{cases} -1 & \text{for a wave moving in the negative x-direction} \\ 1 & \text{for a wave moving in the positive x-direction} \end{cases}$$

- The **amplitude** of each wave is based on an initial amplitude, chosen decay constant, and reflectance coefficient. Each wave is subjected to exponential decay in the form

$$A(t) = A(t - dt)(1 - rdt)$$

  And, for every reflection at the ends, the amplitude is decreased according to the reflectance coefficient such that

$$A(t) = R \cdot A(t - dt)$$

- Finally, the **positions** are determined based on a fixed wave speed which in a real pump would be a result of the material properties, fluid properties, and transmural pressure. The position is therefore

$$\text{pos}(t) = \text{pos}(t - dt) + \text{dir } cdt$$

  In the event of a reflection at a site x, the position is adjusted according to

$$\text{pos}(t) = -(\text{pos}(t - dt) + \text{dir } cdt) + 2x$$

A Gaussian waveform is applied about the calculated positions of all the waves and is reflected in the same manner as the wave position if it crosses a reflection site.

$$P(x) = e^{-\frac{2x}{p}^2}$$

All of the waves are then summed along the length of the line divided discretely into steps of length dL to form a spatial wave profile for each time step. The difference in

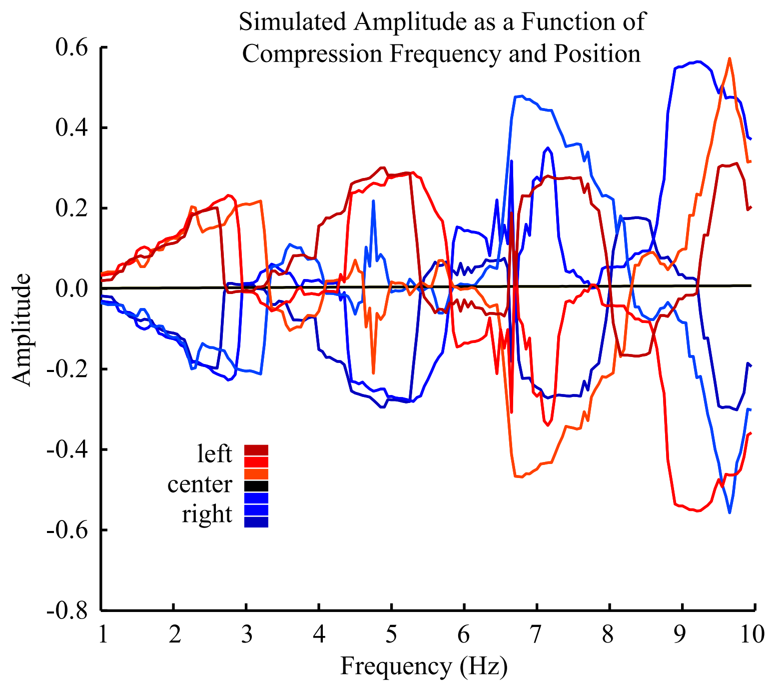


Figure 4.2: Simulation of wave amplitude difference as a function of compression frequency and position.

the summed wave amplitude at the ends of the line represent a value proportional to the pressure head of a similar pump. Once an equilibrium is reach a mean value of the amplitude difference can be taken for varying compression frequencies.

The model was implemented using C++ code. We find similar results to those of the experiments: the frequency response in the time-averaged amplitude difference across the length of the model shifts with the position of compression and is symmetric about the center (figure 4.2); and the frequency response increases linearly with the wave speed (figure 4.3). Additional results from the simulation show that the pulse width greatly affects the amplitude of the difference across the length of the pump (figure 4.4). The reflectance coefficient induces a similar behavior. As the reflectance coefficient increases, so does the amplitude difference across the length. As the reflectance coefficient goes to zero, no net difference is found across the pump (figure 4.5).

A comparison can be made between the experimental results and the simulation

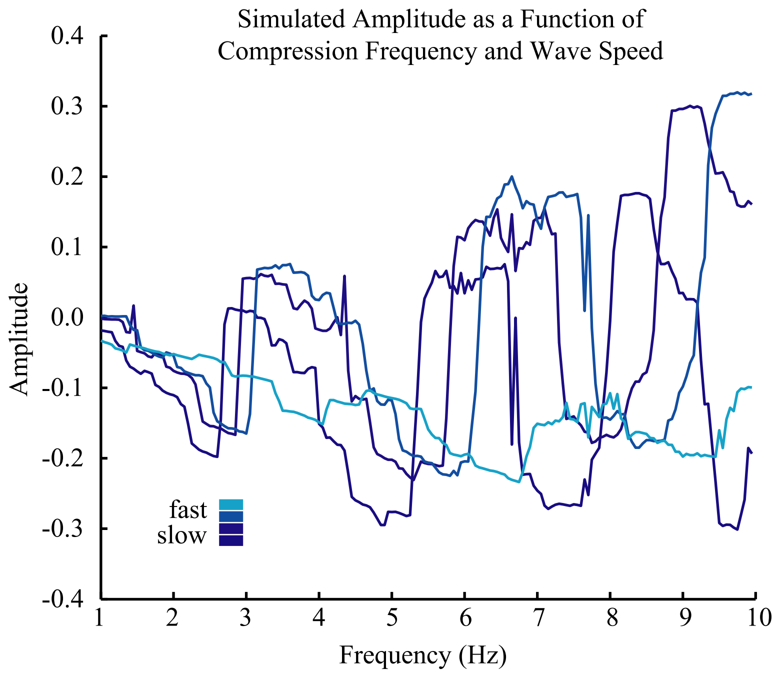


Figure 4.3: Simulation of wave amplitude difference as a function of compression frequency and wave speed.

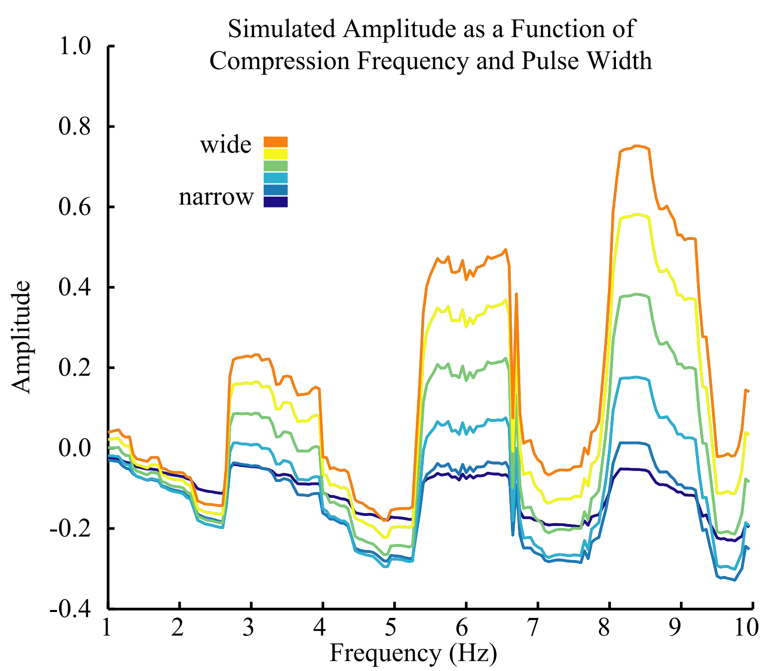


Figure 4.4: Simulation of wave amplitude difference as a function of compression frequency and pulse width.

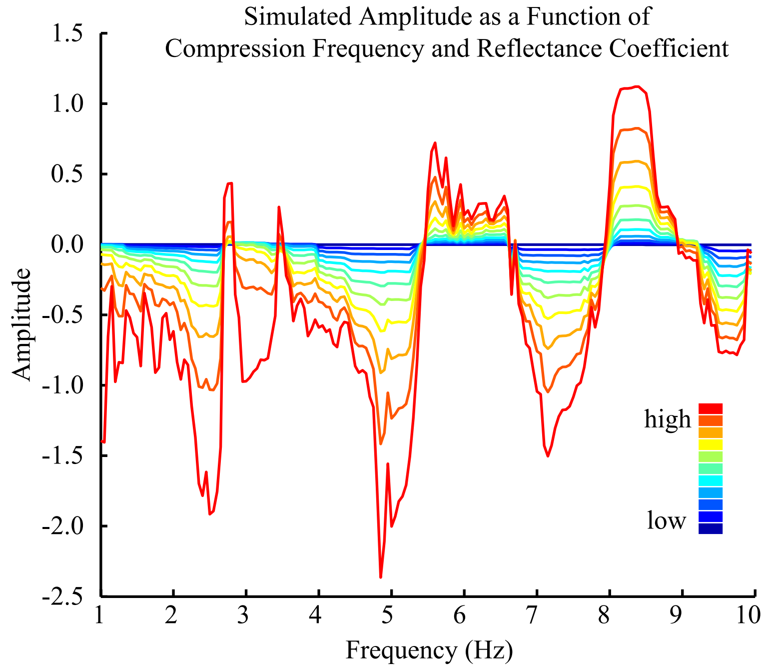


Figure 4.5: Simulation of wave amplitude difference as a function of compression frequency and reflectance coefficient.

using parameters in the range known to be accurate for a specific experiment (table 4.4). Many of the parameters are easily applied from the experiments. To maintain convention used in the experiments, duty cycle is the fraction of the compression period that the pinchers are not in contact with the tube. As was discussed in section 3.2.1, the pinchers only remain in contact with the tube during the compression and not retraction when the motorized compression mechanism is used and the duty cycle had been adjusted accordingly. The parameters that remain unknown are the amplitude of the pressure wave, the waveform including its shape and width, the amplitude decay constant, and the reflectance coefficient. If we do not concern ourselves with the scale of the simulated results, but instead we worry just about the shape, we can safely select the initial wave amplitude, pulse width, and reflectance coefficient without affecting the overall shape. This leaves the shape of the waveform and the amplitude decay constant up to interpretation. The shape chosen was a simple Gaussian loosely based on the ultrasound images of the tube wall. The decay constant was also loosely based the ultrasound images, then further refined to fit the experimental results with the simulated results as closely as possible.

Parameter	Experiment	Simulation
Total time	10 sec	10 sec
Time step, dt	NA	10 msec
Wave speed, c	60 ± 10 m/sec	50 m/sec
Length of tube, $2L$	15 cm	15 cm
Length step, dL	NA	0.01 cm
Compression location, l	-5.1 cm	-5.1 cm
Width of compression, w	2.5 cm	2.5 cm
Duty cycle, d	70%	70%
Initial wave amplitude, A_0	unknown	3
Waveform, $P(x)$	unknown	3 cm wide Gaussian
Amplitude decay constant, r	unknown	0.2
Reflectance coefficient, R	unknown	0.5

Table 4.4: Parameters used for comparing experimental and simulated results.

The simulated results maintain most of the characteristics found in the experiments. There are distinct, sharp peaks at select compression frequencies. Those peaks lie at approximately the same locations. However, it appears that a linear term, if added to the simulated results, would create a more accurate model of the experiment. Capturing this effect will require further modeling. The model can be extended by incorporating dispersion in the form of a time variant waveform. Additional work is necessary to incorporate the interaction with the fluid. For this part, we can borrow from the lumped model techniques and add resistance and impedance. Furthermore, in the real experiments, there is a maximum input that can be exerted on the tube. If the tube is already collapsed at the location of the compression, no work is done if it is compressed again at that time. The disparity between the simulated and experimental results caused by this effect grow with the frequency of compression.

What remains unique and quite exciting about this model is that it begins with wave propagation as its mechanism. It shows that standard wave propagation and wave reflection can be the mechanisms that build a net pressure across the length of the pump. The force causing the wave propagation and the mechanism forcing the reflections are secondary.

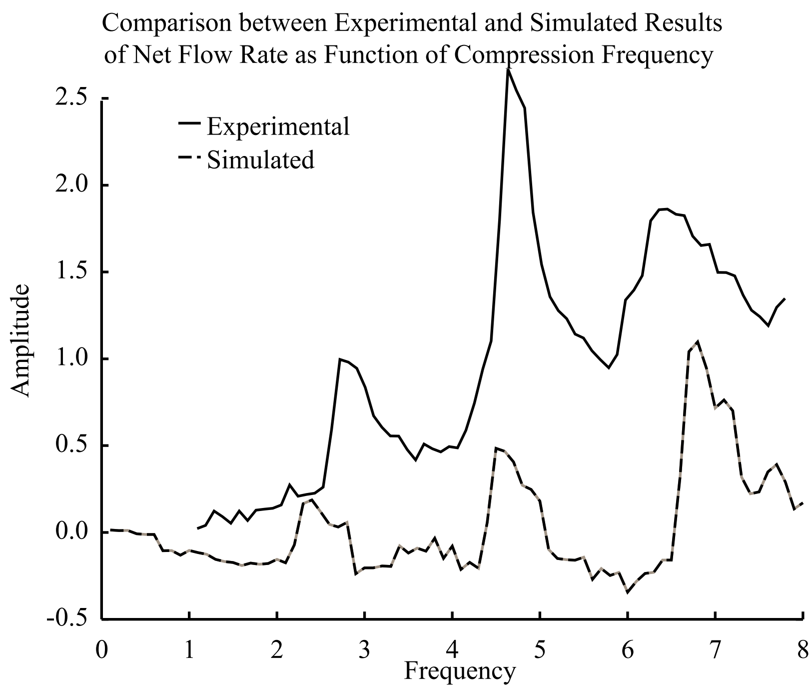


Figure 4.6: Simulation of wave amplitude difference as a function of compression frequency compared with experimental results.